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AND

PROCEEDINGS OF THE SOCIETY OF ARTS.

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PROCEEDINGS OF THE SOCIETY OF ARTS.

THIRTY-SIXTH YEAR, 1897-98.

THURSDAY, April 14, 1898.

THE 511th regular meeting of the SOCIETY OF ARTS was held at the Institute this day at 8 P.M., Professor Swain in the chair.

The record of the previous meeting was read and approved. A paper on "The Exact Estimation of the Total Carbohydrates in Acid Hydrolyzed Starch Products," by G. W. Rolfe and W. A. Faxon, was presented by title, also a paper on "The Reliability of the Dissociation Values Determined by Conductivity Measurements," by Arthur A. Noyes.

The Chairman then introduced Mr. S. Albert Reed, of New York, who read a paper on "Fireproof Construction." The various methods of rendering buildings resistant to fire were described, and then an account was given of tests made by the Underwriters' Association of New York. These showed that iron columns would give way at a dull red heat. The speaker discussed the effects of several large fires, and spoke of the danger of extreme height. The paper was illustrated by the lantern. A discussion followed, after which the Chairman thanked Mr. Reed in the name of the Society, and the meeting adjourned.

THURSDAY, April 28, 1898.

The 512th regular meeting of the SOCIETY OF ARTS was held at the Institute at 8 P.M., Mr. Blodgett in the chair.

The record of the previous meeting was read and approved. The following persons were duly elected Associate Members of the Society: Messrs. Charles H. Fish, of Dover, New Hampshire, A. S. Garfield, of Paris, France, and Walter E. Spear, of Lawrence.

The Chairman then introduced Mr. Albert Sauveur, of Boston, who read a paper[1] on the "Constitution of Steel Considered as an Alloy of Iron and Carbon." Steel was defined as an alloy of iron and the carbide of iron, Fe_3C. Its structure follows the law of that class of binary compounds which do not form definite compounds nor isomorphous mixtures. The curve of freezing points of a saline solution of varying density was compared with the curve of melting points of an alloy of silver and copper of varying proportions, and this in turn was compared with a similar curve of steel containing various amounts of carbon. The three curves were found to be essentially similar, and the eutectic alloy of silver and copper, which is the one having the lowest melting point, and which melts at a constant temperature, was shown to have properties corresponding with those of the cryohydrate of the saline solution on one hand, and with those of pearlite in steel on the other. Pearlite, like the eutectic alloy, is composed of very fine plates of the two substances, alternating in close juxtaposition, and where steel is cooled slowly it forms the matrix containing larger masses of iron (Ferrite = Fe) or of the carbide (Cementite = Fe_3C, containing 6.67% C). When steel is cooled suddenly, however, the matrix is Martensite, the composition of which varies with the proportion of carbon in the steel, and the temperature at which it was quenched.

A discussion followed, after which the Chairman thanked Mr. Sauveur for his very interesting paper, and the Society adjourned.

THURSDAY, May 12, 1898.

The 36th annual meeting (513th regular meeting) of the SOCIETY OF ARTS was held at the Institute on this day at 8 P.M., with Mr. Blodgett in the chair.

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[1] Published in full in the TECHNOLOGY QUARTERLY, June, 1898, Vol. XI, No. 2, pp. 78–92.

The record of the previous meeting was read and approved.

Messrs. C. F. Baker, of Boston, and James G. Melluish, of Bloomington, Illinois, were duly elected Associate Members of the Society.

The annual report of the Executive Committee was read and placed on file.

ANNUAL REPORT OF THE EXECUTIVE COMMITTEE

Presented at the Thirty-sixth Annual Meeting of the Society of Arts, May 12, 1898.

The first meeting of the SOCIETY OF ARTS during the present year was held on October 28, the meeting of October 14 having been omitted to allow members to attend the Memorial Meeting in Music Hall, in honor of President Walker. In all, twelve meetings have been held, and the following papers have been read: "Recent Work in Heat Measurement at the Institute," by Mr. Charles L. Norton; "Cable and Underground Electric Roads," by Mr. Louis J. Hirt; "Contributions to our Knowledge of the Micro-organisms and Sterilizing Processes in the Canning Industries. II. The Souring of Sweet Corn," by Messrs. S. C. Prescott and W. Lyman Underwood; "The Concentration of Ores, with Special Reference to Recent Investigations in this Subject," by Professor R. H. Richards; "A New System of Pneumatic Dispatch Tubes," by Mr. B. C. Batcheller; "The History and Present Development of the American Bicycle," by Dr. Leonard Waldo; "On Town, State, and City Boundaries," by Mr. Henry B. Wood; "Certain Sanitary Aspects of Jamaica," by Mrs. Ellen H. Richards and Mr. Arthur T. Hopkins; "On Fireproof Construction," by Mr. S. Albert Reed; "On the Constitution of Steel Considered as an Alloy of Iron and Carbon," by Mr. Albert Sauveur; and on "The Economic Relation of Deep Inland Waterways to the State of New York," by Mr. George W. Rafter.

The membership of the SOCIETY OF ARTS continues to increase, although there have not been added so many new members as during the previous year. At the close of the year 1896–7 the number of life members was 59; five have died since then, leaving the present number 54. The number of associate members a year ago was 311; of these one has died and nine have resigned. These losses are offset by the election during the year of 22 associate members, making the total number 323.

Early in the autumn the Executive Committee lost one of its members by the death of Mr. Thomas Doane, who was distinguished as an engineer and as the founder of the Doane College in Nebraska. He had been a life member of this Society for many years. The other life members who have died during the year are John M. Forbes, Joseph S. Fay, John Lowell, and Theodore Lyman. We have lost one associate member by death, Mr. C. H. Parker, of Cambridge.

A notable event in the history of the SOCIETY OF ARTS was the election of Professor James M. Crafts to be President of the Institute. He was chosen by the Corporation, October 20, and was introduced to the Society as its President at the meeting of October 28. An amendment to the By-Laws of the SOCIETY OF ARTS was adopted by the Society December 9. This amendment provides that three members shall constitute a quorum of the Executive Committee for the transaction of business.

The Board of Publication, which has charge of the TECHNOLOGY QUARTERLY AND PROCEEDINGS OF THE SOCIETY OF ARTS, was reappointed, with the exception of Professor Henry M. Howe, who has removed to New York. Professor Charles R. Cross was chosen in his place. The following members, therefore, constitute the Board of Publication as now organized: Professor W. T. Sedgwick, Chairman, Professor Charles R. Cross, Professor Dwight Porter, A. Lawrence Rotch, Esq., and the Editor, Dr. R. P. Bigelow. From 1880 to 1891 the SOCIETY OF ARTS published an annual volume of "Proceedings." The TECHNOL-

OGY QUARTERLY was founded in 1887 by the students, and was published by a board of editors chosen from the Junior and Senior Classes, William S. Hadaway, Jr., '87, being the first editor-in-chief. Later Mr. James P. Munroe became sole editor, and in 1892 the SOCIETY OF ARTS assumed control of the journal and united to it the "Proceedings." It thus became the official organ of the Institute. By a vote of the Society the Executive Committee was authorized to appoint annually a Board of Publication to exercise a general control and supervision over the QUARTERLY. The active management, however, falls upon the editor, who is appointed by the Executive Committee upon the nomination of the Board of Publication.

During the present year the TECHNOLOGY QUARTERLY has published not only the minutes of proceedings of the Society, and a number of papers at its meetings, but it has also contained a number of original articles which have been presented to the Society only by title. The series of *Results of Tests Made in the Engineering Laboratories* has been continued, as well as the monthly *Review of American Chemical Research.*

Respectfully submitted,
For the Executive Committee,
GEORGE W. BLODGETT, *Chairman.*

The following officers were elected for the year 1898–99: Mr. Arthur T. Hopkins, *Secretary,* and Messrs. George W. Blodgett, Desmond FitzGerald, Edmund H. Hewins, Frank W. Hodgdon, and Charles T. Main, *Members of the Executive Committee.*

Mr. N. M. Lowe offered the following resolution, which was adopted:

The SOCIETY OF ARTS desires to acknowledge its appreciation of the untiring and efficient labors of its Secretary, Dr. Robert P. Bigelow, and to express its regret that he finds it impossible longer to serve the Society in that capacity. Therefore *Resolved,* that the foregoing minute be adopted by the Society and spread upon the records.

The following papers were presented by title: "Experimental Study of Transformer Diagrams," by H. E. Clifford, N. Hayward, and R. Anderson; "On the Telescope-Mirror-Scale Method, Adjustments and Tests," by S. W. Holman.

The Chairman then introduced Mr. George W. Rafter, of Rochester, New York, who read a paper on "The Economic Relation of Deep Inland Waterways to the State of New York." The early history, he said, of the New York state canals is an exceedingly fascinating study, especially when we consider the extraordinary changes which have taken place since the canals were opened in 1825. The great interest in the Erie Canal is due to the position of the State, which is in the exact line along which the produce of the West must

move to the East. The Mohawk Valley furnishes the great opportunity. The history of early attempts to construct waterways by private corporations was sketched, the history of the canals as at present constructed was described, and then an account was given of the projects for making a deep waterway from the Great Lakes to the sea. Two main routes were described, one by way of Oswego and the Mohawk Valley, and the other through the St. Lawrence River and Lake Champlain. Which is the better route is not yet determined, but the one through the St. Lawrence has the advantage of being a continuous down grade, and requiring less than a hundred miles of artificial channel.

Thirty years ago the canals carried annually from six to eight million tons of freight. In 1896 and 1897 they carried only about three and one-half millions, of which three million tons was through freight, while the New York Central Railroad carried about twenty-two million tons. These figures show that the canal is of little value to the state, and the opinion was expressed that the canal should be turned over to the national government. New York was described as the greatest water power state in the Union, being capable of developing one and a half million horse power, and it would gain by abandoning its canals and developing its water power. The speaker closed by a description of a trip across the state along the Erie Canal, illustrated by lantern slides.

The Chairman thanked Mr. Rafter for his interesting paper, and the Society then adjourned.

ROBERT PAYNE BIGELOW, *Secretary.*

THE TELESCOPE-MIRROR-SCALE METHOD; ADJUSTMENTS AND TESTS.[1]

By SILAS W. HOLMAN.

Received May, 1898.

In the telescope-mirror-scale method for measuring small angles, an illusory estimate of the accuracy attained may easily arise through inattention to the requisite adjustments, tests, and corrections. Yet an adequate and well-ordered presentation of them is not to be found. Partial statements are given in text-books on manipulation, but Czermak[2] alone has given a discussion approaching completeness. Even this omits some essential points, and moreover is not arranged in a manner to lend itself readily to the practice of the method.

The following presentation of the subject is designed for the observer who would put the method into direct service for obtaining measurements of a specified or of a determinate accuracy. It therefore not only discusses the several sources of error, but describes the various adjustments and tests in the sequence in which they should ordinarily be made, and gives a numerical measure of the closeness with which each must be carried out to secure a specified precision in the result. Some remarks on the selection of instruments are appended.

The general plan adopted in treating each adjustment or test is as follows: To deduce a general, though usually approximate, expression for the error attending its omission, or preferably for the correction therefor. To deduce therefrom a numerical measure of the closeness with which the adjustment must be made or the test fulfilled, in order to insure a designated precision in the use of the method, so far

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[1] Copyrighted, 1898, by S. W. Holman. Published separately by John Wiley & Sons, 53 East 10th Street, New York. Cloth, 75 cents.

[2] CZERMAK. "Reduction Tables for Readings by the Gauss-Poggendorff Mirror Method." Besides a discussion of many of the errors of the method, this book gives extended tables of corrections and reductions, of much service in long series of observations. The text is in German, French, and English, in three parallel columns.

as that source of error is concerned; and to make certain comments based thereon.

To render the problem definite, the tangent of the angle of deflection of the mirror is assumed to be the desired result of an observation. The deductions are easily adaptable to other less frequently employed functions. The expressions for the errors or corrections are brought into the form of fractional errors or corrections; that is, they are expressed as a fraction of the value of the tangent of the deflection. The correction is of course equal to the error, but has the opposite algebraic sign, a positive error requiring a negative correction, and vice versa. It is more convenient in the present work to deal directly with corrections than with errors. The numerical solutions are computed on the basis of a desired precision of one part in one thousand, or one-tenth of one per cent., in the resulting value of the tangent. The precision discussion based on methods elsewhere stated[1] is sufficiently obvious. As there are some fifteen sources of error, the average effect to be assigned to each consistently with the prescribed limit of 0.001 in the result is $0.001 \div \sqrt{15} = 0.00\ 030$ nearly enough, as several of the errors are usually rendered insignificant. The solutions are also made for a scale of millimeters at a distance of one meter from the mirror, and for a maximum deflection of 500 mm. It will be found by inspection of the results, especially under I, III, XII, XIII, XIV, XV, that 0.1 per cent. is about the limit of accuracy attainable under these conditions.

In the employment of the apparatus absolute values of the tangent of the deflection are sometimes sought, in which case the measurements may be called *primary*. More often, however, only relative values are required, or rather we are concerned as to the accuracy of only relative values of the tangents. The measurements may then be called *secondary*. An example of the secondary use is where the telescope and scale are employed with a reflecting galvanometer to measure currents, and where the "constant" of the apparatus is found by sending a known current and reading the deflection. In such cases any constant fractional errors in the telescopic method enter into the "constant," as well as into subsequent observations, but with opposite signs, so that they are eliminated from the results. It is therefore needful to discuss the errors with respect to both primary and second-

[1] "Precision of Measurements." John Wiley & Sons, New York.

ary use of the method, and relief is thus found possible in the latter from some of the exactions of the former.

When reversals are taken, that is, when the mirror is deflected so that a reading can be made first on one and then on the opposite half of the scale, certain sources of error are reduced by averaging the two deflections. This is usually precluded in practice, however, by such conditions as continual change in the reading, avoidance of delay, etc. Both cases must therefore be discussed.

The desired tangent or other function of the angle of deflection is computed from the observed scale-reading d and scale-distance r as stated under XVI. We will assume that it is found, with due allowance for the approximation involved (cf. XVI), from the expression

$$\tan \varphi = \frac{1}{2} \cdot \frac{d}{r} (1 + u + v + w + \ldots)$$

where u, v, w, etc., are the fractional corrections to the observed tangent to allow for the various sources of error to be pointed out. As d enters as a direct factor in this expression for the tangent, the fractional corrections may be applied directly to d. In fact it is more convenient in deducing the formulæ to find an expression for either δd, the numerical correction to d, or for $\delta d / d$, the fractional correction to d. Thus δd is such a quantity, expressed in scale divisions, that when added to the observed scale-reading d it will give the correct scale reading, as far as the designated source of error is concerned. And $\delta d / d = u$ or v, etc., is this correction expressed as a fraction of the observed reading d. The algebraic sign of any correction may be either determinate or indeterminate. Also it may be noted that as r enters as a factor in the denominator, the corrections u, v, etc., may be applied to or deduced for r instead of d, if desired, but with the difference that the algebraic sign would be reversed. Any corrections which may prove to have a constant value may be included once for all in the numerical constant $1 / 2r$. Constant fractional corrections disappear when merely relative deflections are used, or when the constant is determined by calibration, as above stated. The assumption is made that none of the errors with which we have to deal are in excess of 1 or 2 per cent. Greater ones must be reduced by instrumental rearrangement before they can be determined with sufficient closeness.

The algebraic expressions which will be deduced for u, v, w, etc., are useful in two ways: First, they enable us to compute the correction to be applied to any observed reading; second, they are used in the precision discussion, which shows how closely each correction must be worked out to secure a prescribed accuracy in the value of tan, φ or how close an adjustment is needed in order that the correction may be omitted without introducing more than its due share of error into the result. These will be called *negligible corrections.* Any error of this magnitude produces an effect not more than is admissible on the result, and the effect of one as small as one-third of this amount will be inappreciable.

The results deduced for the telescope-mirror-scale method are in general directly applicable to the mirror method where a beam of light replaces the telescope.

The Procedure.

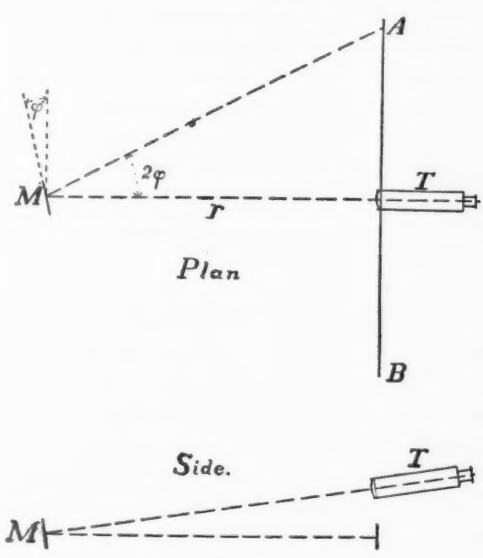

Fig. 1.

Preliminary.—Focus the cross-wires in the eye-piece. Set up the apparatus nearly in proper position as in Figure 1, adjusting telescope, mirror and scale so that the scale is visible. The following description will assume the disposition of the apparatus to be the most usual and simple one, shown in Figure 1: The scale $A\ B$ is horizontal with its middle point just below or above the optic axis of the telescope, which is perpendicular to the scale. The mirror (plane) faces the telescope and scale at a convenient distance. The horizontal distance r between scale and mirror should then

be roughly measured, say to 1 per cent., for use in the approximate computations in the several tests, etc. The selection of this distance is determined largely by convenience, and usually lies between one and three meters. It must be sufficient to insure to the smallest scale-readings to be observed, a sufficient number of divisions to be read with the requisite fractional precision. This means greater distances and higher magnifying power for very small angles. Labor in computation may be saved by making this distance some simple round number, *e.g.*, 1,000 or 2,000 scale divisions, but the work of this adjustment will usually outweigh that of computation. The distance must, however, be maintained accurately constant. The telescope may be at a distance different from that of the scale, if more convenient, but further removal reduces the magnification. As far as the principle of the method is concerned, the telescope may be at any position whatever relatively to the scale, consistent with having the line from the middle of the scale to the mirror nearly horizontal, as pointed out in test X. If there is annoyance from duplicate reflections from the glass front of the mirror house, the glass may be slightly tilted forward.

The following tests and adjustments are then to be made in the order in which they are given, a repetition of any of them being subsequently made if disarrangement may have occurred:—

Adjustments and Tests.

A list is here given of the various adjustments and tests, together with a brief statement for convenient reference of the closeness necessary to correspond with an accuracy of one-tenth of one per cent. in the resulting value of tan φ, assuming a straight millimeter scale at a distance of 1 meter, with deflections not exceeding 500 mm. and not less than 100 mm., readings being taken to tenths of a millimeter by the eye.

I. Cross-Wire Focus. — By parallax test there must be no apparent motion of wires, or less than $\frac{1}{20}$ division. Telescope must also be perfectly focussed. Primary and secondary measurements the same.

II. Optic Axis of Telescope Radial. — P. M. Cross-wires centered within 1 mm., or better, to 0.2 mm., if possible, when tested

on scale at distance of 1 m. Optic axis must pass within 1 mm. of axis of suspension.

III. Scale Graduation. — The average error in the distance of any ruling from the central ruling must be less than 0.03 mm. P. M. and S. M. the same.

IV. Cover-Glass Thickness. — P. M. Glass negligible when less than 1 mm. in thickness. S. M. Glass always negligible. Glass must permit good definition.

V. Cover-Glass Surfaces Parallel. — P. M. To be negligible, the maximum displacement must be less than 0.3 mm. on rotating the glass 180° in its own plane. If more is found, the glass must be kept in the position of minimum displacement. S. M. The same.

VI. Cover-Glass Curvature. — P. M. To be negligible, the focal length of the cover-glass must exceed 70 meters. This will be the case when an object at a distance of 10 meters from the telescope, and sharply in focus with the cover-glass interposed, does not require to be moved through more than about 1 meter towards or from the telescope to maintain the focus when the glass is withdrawn. S. M. The error is here always negligible.

VII. Mirror Thickness. — P. M. Negligible thickness is 0.5 mm. S. M. Always negligible.

VIII. Mirror Eccentricity. — P. M. Negligible up to an eccentricity of 10 mm. S. M. Also negligible.

IX. Mirror Curvature. — P. M. Correction depends on amount of eccentricity. It vanishes with no eccentricity. For specific case see later. S. M. Curvature has no sensible effect.

X. Mirror Vertical and Vertical Angle *TMO* Small. — P. M. Telescope and scale must not be separated vertically by more than 60 mm. S. M. Negligible, and distance need not be very small.

XI. Scale Horizontal. — P. M. Error insignificant when end of scale is not more than 1 or 2 mm. out of horizontal through middle. Negligible up to 12 mm. S. M. Negligible for several centimeters of tipping.

XII. Scale Perpendicular to *OM* and Plane. — P. M. Without reversals, the difference in distance of the ends of the scale (the 500 mm. marks) from the suspension fiber must be less than 0.6 mm. With reversals, 25 mm. is close enough. Scale must be plane within these limits. S. M. About the same as in primary. Hence advantage in reversals where practicable.

XIII. NULL POINT. — P. M. The scale once fixed in place cannot be disturbed without impairing XII. Hence accidental change of null reading must be remedied by turning the mirror, not by moving the scale. With reversals, the null reading may be as great as 17 mm. without correction. Without reversals, the null reading must be less than 0.6 mm., even if allowed for. S. M. Same as in primary.

XIV. DISTANCE OF SCALE FROM MIRROR. — P. M. This must be measured and *kept constant* within 0.3 mm. This demands more than the usual attention, and renders some special device important. See XIV later. S. M. The same.

XV. ESTIMATION OF TENTHS OF DIVISION. — Nearest tenth is to be read in both P. M. and S. M.

XVI. TO COMPUTE THE DESIRED FUNCTION OF q from the observed deflection d. Details are given under XVI later. Five places of significant figures should be used in d, etc.[1]

I. CROSS-WIRE FOCUS.

The cross-wire intersection must be brought accurately into the focus of the eye-piece on each occasion of use of the apparatus, and by each observer for himself. Inattention to this point may easily give rise to an error as great as half a scale division. The focussing should be done by the parallax method, as follows: Focus the wires as sharply as possible by moving the eye-lens. Then focus the telescope very carefully on the scale or on some object showing some sharply marked point of reference. Move the head to and fro sidewise, so that the pupil of the eye shall travel from one side to the other of the aperture of the eye-piece. If the wires are not in focus they will appear to move over the scale. If so, refocus them, and then refocus the telescope on the scale. Continue until no apparent motion of the wires is perceptible. Good focussing is promoted by looking away from the telescope frequently; also in some cases by fixing the attention of the other eye on a printed page held at the distance of most distinct vision beside the telescope. The accurate focussing of the telescope is not less important than that of the wires. The error from imperfect focus will be indeterminate in sign and magnitude.

PRIMARY MEASUREMENTS. — The negligible correction will then be $\pm \delta d$, whose magnitude must be such that $\delta d / d = 0.00\,030$ for

[1] Computation Rules and Logarithms. The Macmillan Co., New York.

the smallest value of d to be used. Now deflections of less than 100 divisions are not employed in exact work for reasons shown in XV. Hence substituting $d = 100$ mm. we have $\pm \delta d = 0.03$ mm. The extreme motion of the eye in the parallax test will obviously produce a displacement of the maximum error, and more than double the average error. Hence the focus will be good enough when the maximum displacement by the parallax test is less than about 0.06 divisions, or about $\frac{1}{20}$ of the smallest scale division. It is clear, then, that the utmost attention to this detail is necessary.

Secondary Measurements. — This source of error affects primary and secondary measurements equally.

II. Optic Axis of Telescope Radial.

This adjustment requires two operations: First, centering the cross-wire intersection, that is, bringing it into the optic axis; second, directing the optic axis towards the axis of suspension, *e.g.*, by focussing the wire intersection upon the suspending fiber.

The second operation is obvious enough, but must be repeated from time to time by way of precaution. It is not easily executed when the telescope draw-tube does not permit of focussing on objects as near as the mirror.

The well-known method of centering the wires is briefly as follows: Lay the telescope in a pair of V grooves — temporary wooden ones will suffice. Focus on the scale, or on a few rulings roughly equal to the scale divisions, placed at a distance about equal to TM. Rotate the telescope in the grooves. The wires are centered when the intersection shows no motion on the scale when thus rotated. If adjusting screws are provided, the diaphragm carrying the wires should be centered until the motion is less than 0.1 or 0.2 mm., as demonstrated later. If, as is commonly the case, there is no means of adjustment of the wires, they must be remounted if the extreme apparent displacement on turning through 180° is more than about 1 mm., for work of the accuracy here assigned.

Demonstration. — We will take the simplest case, where the optic axis TF of the telescope is at right angles to the scale AB. If adjustments XII and XIII had been made the relative positions would be slightly different, but the result would be essentially the same as far as this source of error is concerned. Suppose the telescope so

located that TF meets the mirror at c' instead of c, the latter being in the vertical axis of suspension. Let cc' be denoted by z. Then we seek an expression for the fractional correction $\delta d / d$ for the error which z will produce.

If c were at c', as it should be, and were deflected through an angle φ, the observed reading OA would be the true reading d desired. With c at c', the observed reading becomes OA', and $A'A = \delta d$. As $Tc' = r$, we have $\delta d : c'\ c'' = d : r$. But $c'\ c'' = z \tan \varphi = z\ (d / 2r)$ approx. Hence $\delta d / d = zd / 2r^2$ approx., for a deflection on the side considered. For a reverse deflection the correction would obviously be of the same amount, but the opposite sign.

Imperfect centering of the wires, besides misdirecting the optic

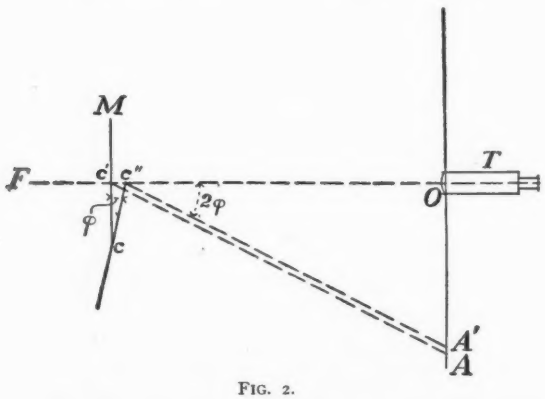

FIG. 2.

axis, introduces a slight error through the smaller magnification of the outer scale divisions, but this may be easily shown to be small compared with the foregoing.

PRIMARY MEASUREMENTS. — *Without reversals.* As the negligible correction above assigned is $\delta d / d = 0.00\ 030$, the negligible value of z for the worst case, where $r = 1{,}000$ mm. and $d = 500$ mm., is to be found from $z = (\delta d / d) / (d / 2r^2) = 0.00\ 030 \times 2 \times (1{,}000)^2 \div 500 =$ 1.2 mm. Thus the optic axis must pass within about $\pm$ 1.2 mm. of the axis of suspension, which demands no inconsiderable care. As the adjustment requires two operations, both must be made closer than this limit, or one must be rendered imperceptible. The latter, by careful centering of the wires, is usually easier of accomplishment.

If this is done so that the displacement in the above test is less than $\frac{1}{3}$ of z, *i.e.*, 0.4 mm. on a scale at the distance TM, or better to 0.1 or 0.2 mm., this part of the error will be negligible.

With reversal of deflections, this source of error vanishes when z does not exceed 2 or 3 mm.

SECONDARY MEASUREMENTS. — Substantially the same degree of care is needed as in primary work.

III. SCALE GRADUATION.

The graduation must be uniform to such an extent that the actual correction δd to any observed deflection d shall be small enough to make $\delta d / d$ less than the assigned limit. The only thorough way to test this is to measure out from the middle point of the scale to each successive smallest division by some comparator or dividing engine, thus determining every value of $\delta d / d$, and then to apply the corrections thus found unless they are negligible. If an error not exceeding $\frac{1}{20}$ mm. is admissible on a mm. scale, the test may be made by laying a standard (Brown and Sharpe) mm. scale against the unknown one, and inspecting the coincidences of the rulings. The first 50 or 100 divisions either way from the middle are not employed in work of even moderate accuracy, because the value of d is then so small that the unavoidable fractional error from the eye estimation and other sources is excessive.

PRIMARY MEASUREMENTS. — The requisite accuracy of graduation for the above assigned limit would be attained when for the smallest value likely to be used for d in such work, viz., $d = 100$ mm., $\frac{\delta d}{d} < 0.00\,030 \quad \therefore \delta d = 0.030$ mm. That is, stated briefly, the average error in the distance of any scale division from 0 should not exceed 0.030 of a division. This limit can hardly be attained in paper scales without great care.

SECONDARY MEASUREMENTS. — Any systematic error of the scale which makes it too long or too short by a constant fractional part, *i.e.*, for which $\delta d / d$ is constant, will be without effect, since it will cause the value of the "constant" found in calibration to be too small or too large by an equal fraction, so that the error in the constant and in deflection readings will exactly offset each other. The *inequalities* of graduation must be, however, less than 0.030 division.

IV. Cover Glass Thickness.

If the glass has plane parallel faces and is of a thickness t, it will displace the ray parallel to itself at each passage. As the simplest case, suppose the glass to be so placed that a ray OM from the center of the scale to the mirror is normal to the glass. Then the ray will experience no displacement in passing from O to M, but would be displaced from MA to gA' in passing from M to the scale. So that with the telescope at O a reading $OA' = d$ would be observed instead of the true reading OA. The correction δd_1 to d_1 is therefore $AA' = gb$. Now $gb = be - ge$, $be = t \tan 2\varphi$, $ge = t \tan gce$. By law of refraction, n being the index of refraction,

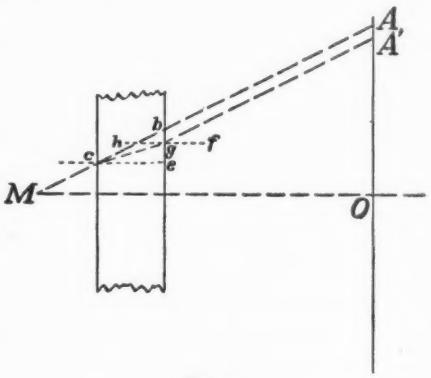

Fig. 3.

$$\frac{\sin hgc}{\sin fgA'} = \frac{1}{n} = \frac{\tan gce}{\tan 2\varphi} \text{ approx., as } 2\varphi \text{ is small.}$$

$$\therefore \delta d_1 = t\,(\tan 2\varphi - \frac{1}{n} \tan 2\varphi) \text{ approx.}$$

$$= t\left(1 - \frac{1}{n}\right) \tan 2\varphi \text{ approx.}$$

For ordinary glass $n = \frac{3}{2}$

$$\therefore \delta d_1 = \frac{t}{3} \cdot \frac{d_1}{r}\,; \therefore \frac{\delta d_1}{d_1} = \frac{t}{3r}$$

Inspection of the diagram shows that the sign of the correction is positive on either side of O. Hence $\frac{\delta d}{d} = \frac{1}{3} \cdot \frac{t}{r}$. The reflecting surface of the mirror is supposed to contain the axis of suspension in all cases unless otherwise specified, so that $OM = r$.

If, however, the glass is not perpendicular to OM, the error becomes unsymmetrical, and dependent upon the horizontal angle between OM and the mirror. It is best, therefore, that the angle should be kept very nearly 90°, as may easily be done. With a very thick cover-glass this point would require special investigation.

The effect of the thickness of the cover-glass may be readily shown from the above formula to be equivalent to the reduction of the scale distance r by one-third of the thickness of the cover-glass.

PRIMARY MEASUREMENTS. — The requisite thinness of the cover-glass, in order that the omission of the correction may be admissible, may be found from

$$\frac{\delta d}{d} = \frac{t}{3r} = < 0.00\,030 \quad \therefore \; t = 3.1000 \cdot 0.00\,030 = 0.9 \text{ mm.}$$

The correction must therefore be applied when, as is almost always the case, the cover-glass is more than about 1 mm. in thickness.

SECONDARY MEASUREMENTS. — Since the fractional correction $\delta d / d$ is constant, the correction disappears in this work as in other similar cases.

V. COVER-GLASS SURFACES PARALLEL.

Defect with regard to this is common. Its effect depends so much upon the various possible angular positions of the surfaces relatively to OM that it is not readily reducible to a general expression, but an easy and sufficient test can be developed. Give the mirror a deflection to about the end of the scale. By clamping, or in some effectual way, hold the mirror so that this reading shall remain fixed; focus sharply; then using due care not to disturb the apparatus, place the cover-glass flat against the objective of the telescope, or better against some diaphragm arranged to hold it not far from the objective; read closely. Now rotate the cover-glass through 180° in its own plane, pressing it against the objective or diaphragm, and read again. A change of reading due to twice the refraction by the prismatic or wedge shape of the glass will be found. Turn the glass in its own plane into various (marked) angular positions and thus locate the diameter along which the displacement is greatest. This will locate the direction of the edge of the wedge, that is, the intersection of the two plane surfaces, since this must be vertical when that diameter is horizontal.

PRIMARY MEASUREMENTS. — It is more prudent not to use a glass which shows a maximum displacement S exceeding 0.3 mm. at a greatest deflection of $d = 500$ mm.; for the negligible correction will be

$$\frac{\delta d}{d} = \frac{1}{2} \cdot \frac{S}{500} < 0.00\,030 \quad \therefore S = 0.3 \text{ mm.}$$

If this limit is not easily reached, the wedge axis may be placed horizontal, which will sensibly eliminate the error. But this requires that during all use of the apparatus the cover-glass be continually inspected to see that it is in the proper position.

In making this test the cover-glass may often, to good advantage, be rotated in position instead of being removed and placed in front of the objective. In that case special care must be taken not to disturb the mirror when the glass is being rotated through 180°. The maximum displacement observed in this method of test in either case is in excess of the worst error which the wedge would cause in the observations, unless it was reversed in position from time to time — which must be guarded against if the glass is poor.

SECONDARY MEASUREMENTS. — No relaxation from the foregoing requirement is admissible.

VI. COVER-GLASS CURVATURE.

Minor irregularities in the surface of the cover-glass of the mirror house produce merely a blurring of the image, such as is seen in looking through ordinary window glass with a telescope. The cover-glass must be sensibly free from this. If either surface of the cover-glass is systematically curved, the glass will act as a lens. The focus of points seen obliquely through the glass is then changed in both distance and direction. The change in distance is either unnoticed or is corrected by the focussing of the telescope. The change of direction causes an error in d. The glass may be equivalent to a spherical lens or to a cylindrical lens. The effect of the change in direction may be studied by the central ray of any beam; we are concerned here with the curvature as revealed by a horizontal section only. For simplicity, suppose the glass to be equivalent to a spherical lens $D\,G$ of focal length f with its axis coincident with $O\,M$. If the lens is convex, the true reading A will be shifted to A' toward O; if concave, in the opposite direction.

The correction required and the method of test may be developed as follows: The lettering of the diagram (plan) being as before, let

$O A = d$ = true deflection reading,

$O A' = d_1$ = observed deflection reading,

$M G = e = p_1$ = distance of cover-glass G from mirror (roughly),

J and M = conjugate foci of lens G when $M G \gtrless f$, the focal length of G,

$J G = p_2$, $A M$, $A' J$ = straight lines.

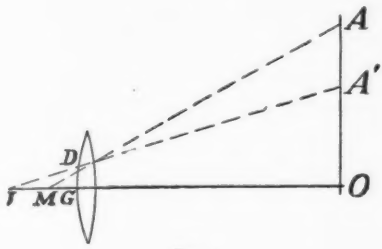

FIG. 4.

As we have treated separately the thickness of the glass, $A M O = a$, $A' J O = \beta$.

$$\frac{\tan a}{\tan \beta} = \frac{p_2}{e} = \frac{d}{r} \cdot \frac{p_2 + r - e}{d_1}$$

$$\therefore \frac{d_1}{d} = \frac{e}{r} + \frac{e}{p_2} - \frac{e^2}{p_2 r}$$

But by the law of lenses, inserting for convenience the negative sign because $p_1 < f$,

$$\frac{1}{p_1} - \frac{1}{p_2} = \frac{1}{f} \text{ or } \frac{1}{e} - \frac{1}{p_2} = \frac{1}{f}.$$

Hence multiplying by e and transposing $\frac{e}{p_2} = 1 - \frac{e}{f}$,

$$\therefore \frac{d_1}{d} = \frac{e}{r} + 1 - \frac{e}{f} - \frac{e}{r} + \frac{e^2}{rf} = 1 - \frac{e}{f}\left(1 + \frac{e}{r}\right).$$

The signs will obviously be the same for deflections on the opposite side of O. The correction is therefore

$$\frac{\delta d}{d} = \frac{d - d_1}{d} \text{ approx.} = 1 - \frac{d_1}{d} = \frac{e}{f}\left(1 + \frac{e}{r}\right).$$

But in all ordinary cases $e < \frac{1}{10} r$, so that for the present purpose $\frac{e}{r}$ is negligible compared with 1, and $\frac{\delta d}{d} = \frac{e}{f}$.

This evidently applies to deflections in either direction, provided that the axis of the lens coincides with OM; otherwise the correction would be unsymmetrical and would contain another term. The effect of this want of symmetry, however, would be detected under test V for wedge shape of cover-glass, and therefore need not be here discussed.

If the cover-glass proves to be cylindrical in the test given later, the worst effect of the cylindrical surface will be produced when its axis is vertical. The correction is then the same as for the spherical surface.

PRIMARY MEASUREMENTS. — The negligible correction may be found from $\frac{\delta d}{d} < \frac{e}{f} = 0.00\,030$.

Assuming a value of $e = 20$ mm., not often exceeded, this yields $f = 20/0.00\,030 = 7.10^4$ mm. $= 70$ m. Hence the cover-glass, whether equivalent to a spherical or a cylindrical lens, must have a focal length exceeding 70 m. How the actual value of f is best determined will presently be shown.

SECONDARY MEASUREMENTS. — The fractional correction to the deflection is constant. Hence in secondary work the error introduced in the "constant" by neglect of the correction exactly offsets the errors entering into the subsequently observed deflections by the same cause, so that the systematic curvature has no effect.

MEASUREMENT OF f. — Direct the telescope (any good telescope other than that of the apparatus will answer if more convenient) upon any well defined object, such as a printed page, held at a distance of several meters (10 or more if practicable). Interpose the cover-glass, placing it directly in front of the telescope. Focus sharply; remove the cover-glass. If the glass is equivalent to a lens with spherical surfaces, the object will cease to be in focus. Without changing the focus of the telescope, bring the object towards it (concave) or move it away (convex) until the focus is again sharp. Let a denote the distance from glass to object in the first position and b in the second. Then $\frac{1}{b} - \frac{1}{a} = \frac{1}{f}$, as the two positions are conjugate foci. In actual measurements it is, of course, better to measure $(a - b)$ directly with some care, and then a roughly (or b), thence computing b and f. To detect unequal curvature of different parts of the glass, it is well to measure f for each of its four quarters successively, covering its remaining surface.

It may sometimes be better merely to test whether f is sufficiently great without actually measuring it. Thus if the limit is $f > .70$ m., then if the object be set up with $a = 10$ m., for example,

$$\frac{1}{b} = \frac{1}{70} + \frac{1}{10} = \frac{80}{700} \quad \therefore\ b = 8.8 \text{ m.}$$

or $a - b = 1.2$ m.

If, therefore, b were distant from a by as much as 1 m., the glass would be good enough. Cover-glasses should never be found so poor as this on good instruments, but this cannot be relied upon, as makers frequently send out very unfit glasses. The glass should be tested by focussing through it separately on vertical, horizontal, and oblique lines. Difference of focus shows a cylindrical form which may or may not be superposed upon the spherical. The test should be applied to that value of f which makes $a - b$ the greatest.

VII. Mirror Thickness.

The mirror usually consists of a thin plane-parallel piece of glass "silvered" on its rear surface. If in any case the mirror were very thick it would require special investigation as to corrections for want of parallelism or planeness of faces; but with thin mirrors ordinarily used, and the quality which is insured by the requirement of good definition, these points may be safely neglected.

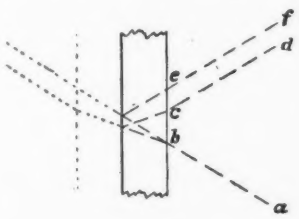

Fig. 5.

Inspection of the figure will show that a ray $a\,b$ passing obliquely into the front surface of the mirror will pass out along $c\,d$, parallel to $e\,f$, the path it would have travelled had there been no glass in front of the reflecting surface. Also, the emergent ray $c\,d$ is displaced from $e\,f$ by the same amount it would have been had it passed through a plane-parallel glass of twice the thickness of the mirror. Since by the process of adjustment the line $O\,M$ is sensibly normal to the mirror, the correction will be symmetrical with respect to O. If, then, t' represents the mirror thickness, the correction to be applied to it will obviously be $\frac{\delta d}{d} = \frac{2}{3} \cdot \frac{t'}{r}$.

PRIMARY MEASUREMENTS. — Requisite thinness of mirror to be negligible = 0.5 mm.

SECONDARY MEASUREMENTS. — The fractional correction being constant may be wholly neglected.

VIII. MIRROR ECCENTRICITY.

If the vertical axis of suspension does not lie in the reflecting surface of the mirror, the eccentricity will cause an error in the scale reading. As the simplest case, let $c\,M$ represent the reflecting surface (plane) in its normal position, $O\,M$ being perpendicular to it; and let Y be the axis of rotation. When the mirror turns through an angle φ to the position M', the reflected ray going to the telescope at O will be $A'\,N$ instead of $A\,N$ which it would have been had Y passed through M. The correction to the observed reading $d' = O\,A'$ is therefore $A\,A'$; and for this an expression must be found. From the figure,

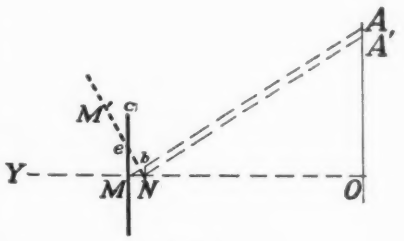

FIG. 6.

$$\delta d_1 = A\,A' = b\,N = M\,N \cdot \tan 2\varphi,$$

$$M\,N = e\,M \cdot \tan \varphi = \frac{1}{2} c\,M \cdot \tan \varphi = \frac{1}{2}\ Y\,M \cdot \tan^2 \varphi,$$

$$\tan \varphi = \frac{1}{2} \tan 2\varphi \text{ (approx.)} = \frac{1}{2} \cdot \frac{d_1}{r}. \quad \text{Also let } Y\,M = X,$$

$$\therefore\ \delta d_1 = \frac{X}{8} \cdot \frac{d^3}{r^3}.$$

This correction will obviously be the same in sign and amount for equal deflections in either direction. Hence

$$\frac{\delta d}{d} = \frac{1}{8} \cdot \frac{X}{r} \cdot \frac{d^2}{r^2}.$$

PRIMARY MEASUREMENTS. — The requisite smallness of X may be found from

$$\frac{\delta d}{d} = \frac{1}{8} \cdot \frac{X}{r} \cdot \frac{d^2}{r^2} < 0.00\,030,$$

$$\therefore\ X = \frac{8 \cdot 1000 \cdot 1000^2}{500^2} \cdot 0.00\,030 = 10 \text{ mm}.$$

It is rare that the arrangement of the apparatus is such that the eccentricity exceeds a few millimeters, so that this correction is ordinarily quite negligible. If the normal position of the mirror is oblique instead of perpendicular to OM, the correction becomes unsymmetrical, and would require further investigation for special cases if the eccentricity were large.

If the apparatus is such that the correction is not small and must be applied, the above form may be insufficient, owing to the inaccuracy produced by the approximation $\tan \varphi = \frac{1}{2} \tan 2\varphi$ employed in deducing it. It is, however, easy to modify the expression so that it may have any desired accuracy, by inserting in place of the approximation the series expressing $\tan \varphi$ in terms of d/r given later.

Secondary Measurements. — As the fractional correction to d has a constant value it vanishes for all values of d as in former cases.

IX. Mirror Curvature.

This produces no error if the axis of suspension of the mirror is tangent to the spherical reflecting surface. If the curvature of the mirror is irregular, the definition will be impaired; but if the telescope collects the light from the whole mirror, no sensible irregularity of reading will be caused.

In the diagram, which is exaggerated for clearness, let Y be the vertical axis of suspension, MR the mirror undeflected (assumed concave), and OY the line from Y to the center of the scale, and to which the undeflected mirror will be normal. Suppose the mirror to be turned through an angle φ about Y, then its new position will be $M'R'$, M having moved to M' through the angle $MYM' = \varphi$. $YM'C$ is now normal to the mirror, and let $M'C$ be the radius ρ of curvature of the mirror. A ray along OY would now meet the mirror at E, whereas, if the

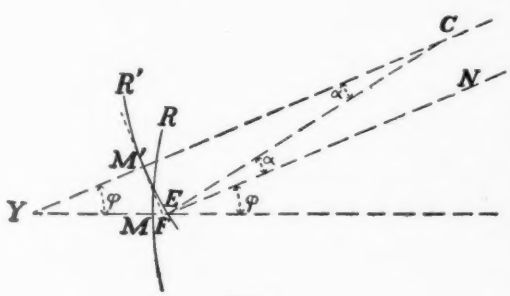

Fig. 7.

mirror had been plane, the reflection would have been from F; and if there had been no eccentricity, from M, since Y would then have passed through M. The error due to the forward motion from F to E would be capable of correction by the same expression as that for MF, but it may be shown to be so small as to be negligible except when ρ is very short, in which case, however, the mirror is not safe for use in accurate work even with the application of the correction. It is therefore at first necessary to deal only with the error which comes from the increased angle between the normal and OE. Owing to the curvature of the mirror, the normal to the mirror at E, instead of lying parallel to YC and therefore making an angle φ with OE, makes with it a greater angle $CEO = \varphi + \alpha$, where α is of course equal to the angle between the tangents at M and M'.

We desire an expression for the fractional correction $\delta d / d$. But

$$\frac{\delta d}{d} = \frac{\tan 2\varphi_1 - \tan 2\varphi}{\tan 2\varphi_1} = \frac{\tan 2\varphi_1 - \tan 2\varphi}{\tan 2\varphi} \text{ approx.},$$

$$= \frac{\tan \varphi_1 - \tan \varphi}{\tan \varphi} \text{ approx.}$$

Now $\varphi_1 = \varphi + \alpha$, and α is always very small;

$$\therefore \tan \varphi_1 = \frac{\tan \varphi + \tan \alpha}{1 - \tan \varphi \tan \alpha} = \tan \varphi + \tan \alpha \text{ approx.}$$

$$\therefore \frac{\delta d_1}{d_1} = \frac{\tan \alpha}{\tan \varphi}.$$

But as $M' \therefore C$ and EN are parallel, $M'CE = \alpha$; and as E and F are nearly coincident and FM' is perpendicular to YC,

$$\frac{\tan \alpha}{\tan \varphi} = \frac{X}{\rho},$$

$$\therefore \frac{\delta d_1}{d_1} = \frac{X}{\rho} \text{ approx.}$$

For a deflection in the opposite direction the correction would be the same, so that the general expression for the correction is

$$\frac{\delta d}{d} = \frac{X}{\rho}.$$

For a concave mirror ρ is $+$, for a convex mirror $-$.

Primary Measurements. — The limiting value of ρ may be found from

$$\delta d_1 / d_1 \lessapprox 0.00030 = X / \rho.$$

With $X =$ zero, that is, no eccentricity, the correction vanishes. For $X =$ 1 mm., $\rho =$ 3300 mm. $=$ 3.3 m. For $X =$ 10 mm., $\rho =$ 33 m. and so on. An accidental curvature of less than 3 m. is not unlikely, and an eccentricity of 1 mm. is not uncommon and by no means always avoidable. It is therefore necessary to measure ρ roughly as shown below, but it is unlikely that the correction will be large when a presumably plane mirror is used, and a little consideration of the character of the error will show that a mirror requiring a large correction cannot be employed.

Secondary Measurements. — As the fractional correction is constant, it may be entirely omitted.

To Measure ρ. — Focus the telescope sharply on the reflection of the middle of the scale from M as in using the apparatus. Measure MO within a few mm. Turn the telescope slightly so as to be able to look beyond the mirror, and place a printed page or other suitable object in the line of sight at a distance about equal to MO. Without changing the former focus of the telescope, move the object towards and from the telescope until a point P is found at which the focus is again sharp. Measure MP within a few mm. Then the radius of curvature of the mirror is

$$\rho = 2 \cdot \frac{MO}{1 - \frac{MO}{MP}}$$

in which the numerical values of both MO and MP are considered positive. If $MP > MO$, clearly ρ is positive and the mirror is concave. If $MP < MO$, ρ is negative and the mirror convex.

If it is merely necessary to know whether ρ exceeds a specified limit, this expression may be transformed into

$$MP = \frac{MO}{1 - \frac{2\,MO}{\rho}}.$$

It is then merely necessary to calculate the value of MP and to see that the object is in focus at a distance less than this in the above test.

X. Mirror Vertical and Vertical Angle TMO Small.

Let $MH =$ horizontal line through M,

$MN =$ normal to mirror,

$TMO =$ line of sight when $q = 0$.

The reflecting surface of the mirror is here assumed to coincide with the axis of suspension, so that $M H = r$.

If the mirror is vertical and the telescope and scale are both at H, no correction will be required. The mirror may be made vertical or nearly so, but T must usually be either above or below the scale, hence the angle $T M O$ cannot be zero. Figure 8 represents the case when neither M nor $T M O$ are zero, and where $N O < N T$. For this case the fractional correction will be shown to be

$$\frac{\delta d}{d} = \frac{N H \cdot N O}{r^2}.$$

This obviously approaches zero as $N H$ approaches zero, *i. e.*, as M becomes more and more nearly vertical. It will disappear if the mirror is exactly vertical whatever the angle $T M O$ (within the limits for which the formula holds). But as it is of course impossible to render M exactly vertical, it is necessary to inquire how nearly so it can probably be rendered, and what would be the corresponding limit of $T M O$. With considerable care the mirror may be made so nearly vertical that N will fall within 10 mm. of H.

Inserting then $N H = 10$, $r = 1000$, and $\delta d / d \leqq 0.00030$ and solving for $N O$ gives $N O = 0.00030 \cdot 1000^2 \div 10 = 30$ mm.

Primary Measurements. — Thus as $N O$ is approximately equal to $N T$, the telescope and scale must not be farther apart than 60 mm. Many forms of telescope and scale are so faulty in design that this closeness of approach is impossible. Most forms provide for motion over a much greater range. No such motion is necessary, and none is desirable.

Secondary Measurements. — Since the correction is a constant one so long as T, N, and O remain fixed, no correction is necessary in secondary work, and no special care to have $N H$ and $N T$ small. Since, however, the correction formula is only approximate and applies only to small values of $N O$, the adjustment should be made somewhat nearly to the above limit. The values of $N H$ and $N T$ must not change during a series of measurements by as much as the above amounts.

Demonstration. — Let Figure 8 represent the apparatus in side view, M being the mirror, O the middle point of the scale, T the telescope, $M N$ the normal to the mirror when deflected, and $M H$

a horizontal line through M. Let the dotted line through O be a vertical line, and assume T, N, and H to lie upon that line, so that all the lines of the figure lie in a vertical plane through $O\,M$. The mirror, which should be vertical, is inclined to its vertical axis of rotation by the small angle a. Thus the normal $M\,N$, which should coincide with $M\,H$ when undeflected, makes with it the angle $N\,M\,H = a$. The telescope and scale should coincide (T with O), but cannot. Hence the telescope is at any position T above or below the scale O, and $N\,M\,T = N\,M\,O = \beta$.

Let Figure 9 represent a vertical plane through the scale and viewed along $M\,H$. Then $S\,S$ will represent the scale, $H'\,H''$ a horizontal line through H and at right angles to $M\,H$, and T', N', O', and H' the points T, N, O, and H respectively. Suppose now the

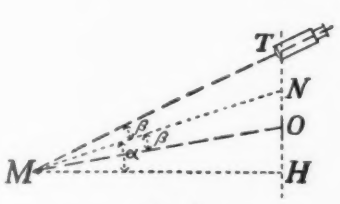

FIG. 8.

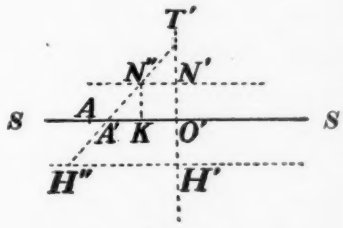

FIG. 9.

mirror is turned about its vertical axis of suspension, assumed to pass through M, through a small horizontal angle φ. Thus the normal $M\,N$ will describe a conical surface with a vertical axis and having its vertex at M. This cone would intersect the vertical plane through the scale in an hyperbola with its vertex at M'. Let $N'\,N''$ be a horizontal straight line; then if $N'\,N''$ be the horizontal projection on this plane of the part of this hyperbola described when M turns through φ, we have,

$$\tan \varphi = N'\,N'' \,/\, M\,H = N'\,N'' \,/\, r.$$

It is also true that as φ is small, the hyperbola will sensibly coincide with $N'\,N''$, but we need not make this assumption. Since the reflected ray will lie in a plane containing T and $M\,N''$, the observed scale reading when M is deflected through φ will be at A', the intersection of a straight line $T\,N''$ (prolonged) with the scale. Since it is read by a horizontal scale having vertical rulings, the distance $O'\,A'$

along the scale will be the horizontal projection upon the scale of the parabolic path of a ray from T reflected by M upon the plane of the scale. This short portion of the parabola of the ray will sensibly coincide with the horizontal straight line $S\,S$.

Let A represent the point (unknown) of the scale at which the true reading corresponding to the observed reading A' would fall; then

$$\delta d_1 = O'\,A - O'\,A'.$$

For values of φ so small that $\tan 2\varphi = 2 \tan \varphi$ nearly enough,

$$\delta d_1 = 2\,r \tan \varphi - O'\,A' = 2\,N'\,N'' - O'\,A'.$$

But also for larger values of φ, within the usual limits, although the value of $\tan 2\varphi$ exceeds $\tan \varphi$ in a continually increasing ratio, the value of $O'\,A'$ increases from the same cause in sensibly the same ratio; so that the above expression for $\delta\, d_1$ holds nearly enough for all cases. From the diagram,

$$O'\,A' = O'\,K + K\,A' = N'\,N'' + N''\,K \cdot \tan O'\,T'\,A' = N'\,N'' + NO \cdot \frac{N'\,N''}{N\,T}.$$

Whence as $2\,N'\,N'' = d_1$ nearly enough (as a factor),

$$\frac{\delta\, d_1}{d_1} = \frac{1}{2}\left(1 - \frac{NO}{N\,T}\right).$$

But,

$$\frac{NO}{N\,T} = \frac{M\,O}{M\,T} = \frac{r\cos(\alpha + \beta)}{r\cos(\alpha - \beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= 1 - 2\sin\alpha\sin\beta \text{ approx. when } \alpha \text{ and } \beta \text{ are small;}$$

$$= 1 - 2\,\frac{NH \cdot NO}{r^2} \text{ approx.}$$

$$\therefore \frac{\delta\, d_1}{d_1} = \frac{NH \cdot NO}{r^2} \text{ approx.}$$

The correction has the same sign on either side of O, hence

$$\frac{\delta\, d}{d} = \frac{NH \cdot NO}{r^2} \text{ approx.}$$

In the last approximation $N\,T$ might have been introduced instead of NO. The choice is determined by the fact that O and N lie nearer

to H than T and N, so that the approximation is closer in assuming $\sin \beta = N O / r$. If, therefore, in an actual case the telescope is much nearer to the horizontal than the scale, it will be slightly better to insert $N T$ instead of $N O$. The sign of the correction to r is — when $N O < N T$ and + when $N O > N T$.

XI. Scale Horizontal.

To adjust the scale horizontal, focus T on the scale, with M swinging free. Note the height of the cross-hair intersection upon the divisions of the scale. Deflect M to the right and left. The intersection should remain at the same height. If it does not, raise one end of the scale.

If the scale be tipped upward or downward from the horizontal through a small angle γ, remaining in the same vertical plane, the reading on either side will be shortened by the versed sine of the angle γ; that is, the fractional correction to d will be

$$\frac{\delta d}{d} = \text{versin } \gamma = 1 - \cos \gamma.$$

Primary Measurements. — For the requisite closeness of adjustment,

$$\frac{\delta d}{d} = 1 - \cos \gamma \leqq 0.00\,030,$$

$$\therefore \cos \gamma = 0.99\,970, \text{ and } \gamma = 1.^{\circ}4.$$

In making the test the observed change in the height h of the cross-hair intersection in passing from the middle to the end of the scale, would be

$$\delta h = 500 \tan \gamma.$$

Hence the requisite closeness will be

$$500 \cdot \tan \gamma = 500 \cdot \tan 1.^{\circ}4 = 12 \text{ mm.}$$

This adjustment can be made with perfect ease to 1 mm. or less, so that this error disappears.

Secondary Measurements. — As $\frac{\delta d}{d}$ is constant so long as γ is constant, the amount of tipping of the scale within wide limits makes no difference so long as it remains always at the same angle.

XII. Scale Perpendicular to *O M* and Plane.

To render *M O A* a right angle, lay off or select two points *A* and *B* near the ends of the scale, exactly equidistant from *O*. Measure carefully the distances *M A* and *M B* from the suspension fiber of *M*. Adjust the scale until the two are equal. When this and the preceding adjustment have been completed, the scale should be rigidly and permanently fixed in place, and means provided to enable the fiber to be always brought back to its present position. A method of support for the scale quite separate from the support of the telescope, as described later, contributes to stability and convenience. It may be noted that inequality of scale readings on reversal of *M* is not a proof of imperfection in this adjustment, nor equality a sufficient proof of correct adjustment.

The fractional correction will be shown to be

$$\frac{\delta d}{d} = \mp \sin \beta \tan 2\varphi - \frac{1}{2} \sin^2 \beta \text{ approx.},$$

where β is the *small* angle *A O A'* by which the scale is out of adjustment. The negative sign of the first term applies to deflections towards *A*, the positive towards *B*, used right-handedly.

Primary Measurements. — If deflections are taken on one side only, *i. e.*, without reversals of *M*, the requisite closeness in β will be attained when

$$\pm \sin \beta \tan 2\varphi = 0.00\,030$$

as $\frac{1}{2} \sin^2 \beta$ is negligible when the apparatus is closely adjusted. For $r = 1000$ mm., $d = 500$ mm.,

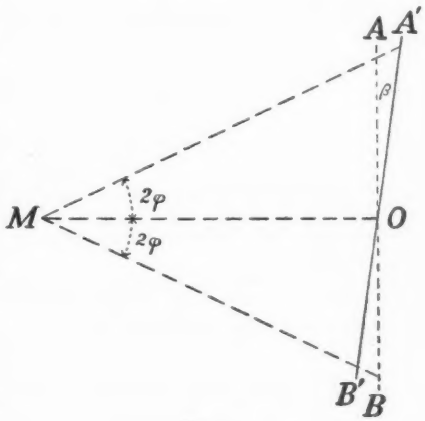

Fig. 10.

$$\sin \beta = 0.00\,030 \frac{1000}{500} \text{ roughly} = 0.00\,060.$$

This corresponds to $\beta^\circ = 0.00\,060 / 0.0174$
$= 0.^\circ035$ roughly $= 2'$.

But it is more convenient to have the requisite closeness expressed in terms of $MA' - MB'$. Now, nearly enough when β is very small, $AA' = BB' = OA \cdot \tan\beta = OA \cdot \sin\beta = 0.00060 \cdot 500 = 0.30$ mm., and also $MA' - MB' = AA' + AB' = 2 \cdot OA \cdot \tan\beta = 0.60$ mm. Hence, the requisite closeness in $MA' - MB'$ *without reversals* is 0.6 mm.

If reversals of deflections are taken, giving d_1 on one side and d_2 on the other, the corrected deflection is

$$\frac{1}{2}\left[d_1 + d_2 - \sin\beta \tan 2\varphi - \frac{1}{2}\sin^2\beta + \sin\beta \tan 2\varphi - \frac{1}{2}\sin^2\beta\right]$$

$$= \frac{1}{2}(d_1 + d_2) - \frac{1}{2}\sin^2\beta.$$

Hence the requisite closeness will be attained when

$$\frac{1}{2}\sin^2\beta' = 0.00030, \quad \therefore \sin\beta' = 0.025.$$

This corresponds to $\beta'^\circ = 0.025 / 0.0174 = 1.^\circ 4$ roughly. Then $2 \cdot OA \cdot \tan\beta = 25$ mm. Hence the requisite closeness in $MA' - MB'$ *with reversals* is only 25 mm. This demonstrates a great advantage in reversals in primary work when practicable. For not only is the scale more readily adjusted, but a slight undetected deviation of it or a slight lateral displacement of the suspension is much less serious.

The scale must evidently be plane within the same limits within which it must be perpendicular to OM.

Secondary Measurements. — In these nearly the same closeness is requisite in $\sin\beta$ as in primary work. If, however, deflections are restricted to the outer half of the scale, *i. e.*, 250 to 500 mm., a little less closeness will suffice, since the error at the deflection reading (whether with or without reversals) at which the calibration is made will enter into the value of the constant, and this will eliminate the error at the *same point* in subsequent readings. It will also reduce the average error about one-half if the calibration deflection is about 350 to 400 mm., but the difference in error of a small and of a large deflection will still remain the same.

Demonstration. — Let A and B be any points on the scale equidistant from O and on opposite sides. Then OA and OB will be the true scale readings with the scale properly adjusted, and will be equal. OA' and OB' will be the observed readings.

The fractional correction $\delta d_1 / d_1$ on the side towards A' will be

$$\frac{OA - OA'}{OA'} = \frac{OA}{OA'} - 1.$$

But $$\frac{OA}{OA'} = \frac{\sin(90° - 2\varphi - \beta)}{\sin(90° + 2\varphi)} = \frac{\cos(2\varphi + \beta)}{\cos 2\varphi}$$

$$= \frac{\cos 2\varphi \cos \beta - \sin 2\varphi \sin \beta}{\cos 2\varphi} \doteq \cos \beta - \sin \beta \tan 2\varphi.$$

Now $\cos \beta = (1 - \sin^2 \beta)^{\frac{1}{2}}$

$$= 1 - \frac{1}{2} \sin^2 \beta \text{ approx., as } \beta \text{ is very small.}$$

$$\therefore \frac{\delta d_1}{d_1} = \frac{OA}{OA'} - 1 = -\frac{1}{2} \sin^2 \beta - \sin \beta \tan 2\varphi.$$

Similarly on the side toward B,

$$\frac{\delta d_2}{d_2} = \frac{OB}{OB'} - 1 = -\frac{1}{2} \sin^2 \beta + \sin \beta \tan 2\varphi.$$

Or, in general,

$$\frac{\delta d}{d} = \mp \sin \beta \tan 2\varphi - \frac{1}{2} \sin^2 \beta.$$

XIII. Null Point.

The "null point" or "zero reading," *i. e.*, the reading when the mirror is undeflected, must be the middle point O of the scale, adjustment XII having been made; and OM must lie in a vertical plane perpendicular to the scale. In other words, the line of sight OM, when M is undeflected, must lie in a vertical plane which is perpendicular to the scale at its middle point O. This does not require either that any normal to M should lie in or parallel to that plane, or that the axis of the telescope should lie in that plane. Or, as less precisely expressed, the plane of the mirror need not be parallel to the scale, nor the axis of the telescope be exactly above or below O. The telescope may be at any position off at one side if more convenient, but ease and accuracy of adjustment, as well as other considerations, lead to the customary location of the axis of the telescope more or less exactly above or below O. Provided that the axis of rotation (suspension) and the reflecting surface of M are sensibly coincident, and that

X, XI, and XII have been completed, this adjustment may be accurately made for any position of the telescope thus: Focus the telescope sharply on the suspension fiber (axis of rotation) of M, with M swinging free. Turn the telescope, or shift it laterally without disturbance of the scale, until the cross-hair intersection falls upon the fiber. Still without disturbing the scale, tip the telescope about a horizontal axis parallel to the scale until the cross-hairs are approximately central on the mirror, and change the focus until the scale is sharply defined. The reading will in general not be exactly at the middle point, but the telescope and scale must now be clamped rigidly in position, and all subsequent adjustment of the null reading to zero must be made *by the mirror*, that is, by changing the direction of the suspended system until the null reading is exactly the middle point O from which A' and B' are measured off in making adjustment XII. This will be effected according to the nature of the apparatus; *e.g.*, by changing the directive field, in a sensitive galvanometer; by turning the torsion head or the whole instrument if it is an electrodynamometer or electrometer, or by twisting the mirror upon its suspension rod, and so on.

If the axis of the suspension and the reflecting surface of M do not coincide, the above method of adjustment becomes inaccurate to an extent depending upon the eccentricity of the reflecting surface, and on the departure of the telescope from the vertical through O. If the eccentricity is but a few millimeters, its effect is wholly negligible if T is within a few millimeters of this vertical, as may be seen from adjustment VIII. In that case no attention to the adjustment of T and O beyond casual inspection is called for.

If when this stage of the adjustment is reached it is found that the cross-hair intersection does not fall at the right height upon the scale for good reading, the telescope may be tipped slightly as a remedy. But if when this is done the illumination or extent of field is not all that the apertures of telescope and mirror should give, then the adjustments must all be repeated, beginning by raising and lowering the telescope and scale together, and tipping the former more or less, focusing centrally on the mirror and then on the scale alternately.

During observations it is by no means always practicable to bring the null reading exactly to the middle point of the scale. This reading d_0, however, is taken before and after each deflection, or with sufficient frequency, and the observed deflection reading d' is corrected

for d_0, or reversed readings are taken, giving the same result. The advantage of numbering the scale from one end continuously to the other instead of both ways from a middle point is apparent in this connection. For in the former case no attention to the sign of either d' or d_0 will be necessary, and corrected deflections, viz., $d' - d_0$ will be always — on the side towards the zero and + on the other side, the sign taking care of itself.

But even with the correction for the observed null reading applied, there remains an error from the fact that the ray $M d_0$, or rather the vertical plane through it, is not exactly at right angles to the scale. The question remains how closely this angle must approach 90° ; or in other words, what is the limit of displacement of the null reading when corrected for, either by observing d_0 or by reversing.

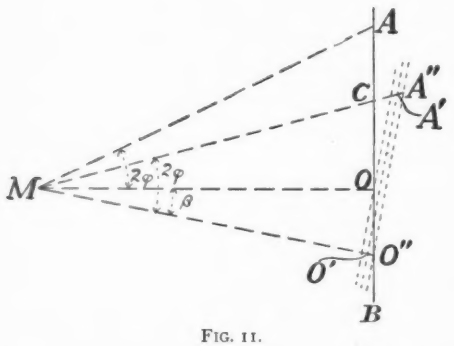

FIG. 11.

Suppose the scale to be properly adjusted at $A B$, and that the null reading becomes subsequently displaced (*e.g.*, by change of torsion in the suspension, of direction of field, etc.) from O to O''.

The observed reading towards A for a deflection φ of M corrected for d_0 ($= O\,O''$) will be $O''\,C$. The true reading would have been $O\,A$. It is desired then to find an expression for the fractional correction to reduce $O''\,C$ to $O\,A$. If a horizontal scale were placed perpendicular to $M\,O''$ at a point O' such that $M\,O' = M\,O$, then the observed reading $O'\,A'$ on such a scale would be equal to the true reading $O\,A$. Imagine also a scale $O''\,A''$ parallel to $O'\,A'$ but drawn through O''. Then

$$\frac{O'\,A'}{O''\,A''} = \frac{O'\,M}{O''\,M} = \frac{O\,M}{O''\,M} = \cos\beta.$$

But as shown under XII (case of $O\,B\,/\,O\,B'$),

$$\frac{O''\,A''}{O''\,C} = \cos\beta + \sin\beta\,\tan 2\varphi$$

$$\therefore \frac{O'A'}{O''C} = \cos^2\beta + \cos\beta \sin\beta \tan 2\varphi$$

$$\therefore \frac{OA}{O''C} = 1 - \sin^2\beta + \sin\beta \tan 2\varphi, \text{ approx.,}$$

as with β very small, the factor $\cos\beta$ is so nearly unity as not sensibly to affect the last term;

$$\therefore \frac{\delta d_1}{d_1} = \frac{OA - O''C}{O''C} = -\sin^2\beta + \sin\beta \tan 2\varphi$$

and in general

$$\frac{\delta d}{d} = \mp \sin\beta \tan 2\varphi - \sin^2\beta,$$

the + and − signs in the first term applying respectively to right and left-hand deflections φ.

Primary Measurements. — *With Reversals of M.* Disregarding the observational sign of d, the mean deflection used would be $\frac{1}{2}(d_1 + d_2)$, and the correction would be

$$\frac{\delta d}{d} = \frac{1}{2}[\sin\beta \tan 2\varphi - \sin^2\beta - \sin\beta \tan 2\varphi - \sin^2\beta]$$

or $= -\sin^2\beta$ for any value of φ.

The requisite closeness of β would therefore be found from

$$\frac{\delta d}{d} \lessdot 0.00\,030 = \sin^2\beta$$

$$\therefore \sin\beta = 0.017;\ \beta = 1.°0, \text{ or}$$

$$O''O = 1000 \tan\beta = 17 \text{ mm.}$$

Without Reversals, the correction would depend chiefly on φ as β would be very small compared with any practicable value of φ. Hence nearly enough,

$$\frac{\delta d}{d} = \mp \sin\beta \cdot \tan 2\varphi.$$

The requisite closeness of β for the worst case where $r = 1000$ mm., $d = 500$ mm., would then be found from

$$\frac{\delta d}{d} \lessdot 0.00\,030 = \pm \sin\beta \cdot \frac{500}{1000}$$

$$\therefore \sin\beta = 0.00\,060 \quad \beta = 0.°035 = 2', \text{ or}$$

$$O\,O'' = 1000 \tan\beta = 0.6 \text{ mm.}$$

Thus with reversals a rather rough adjustment of the null reading from time to time is sufficient; but with deflections on one side only, even with the null reading allowed for, this reading must never exceed about half a millimeter without the application of the above special correction $\delta d_1 / d_1$ or $\delta d_2 / d_2$. It is also essential to note that the adjustments of the null reading must be made by turning the mirror, *not* by shifting the scale.

Secondary Measurements. — The requirements here are precisely the same as in primary work, and the frequent and often unavoidable practice of reading without reversals makes the above remarks of special importance. It is obviously much better to reverse where practicable.

XIV. Distance of Scale from Mirror.

This is invariably *the shortest horizontal distance from the axis of suspension to the vertical plane through the scale rulings.* The reflecting surface of the mirror should lie near to this axis when possible, but the measurement of r must be from the actual suspension horizontally to a vertical line through the middle point of the scale after XII has been performed. It is important to note, as the following pages will show, that most of the adjustments become more easy and the corrections smaller as the scale distance is increased. Therefore, as scale errors are easily reduced, it is better to make r as large (up to 3 or 4 m.) as is consistent with the magnifying power of the telescope, using a scale of not more than 1 m. in length. The telescope, scale, and instrument containing M are set in their proper positions and completely adjusted before r is measured. This is then carefully determined by a horizontal wooden rod, wire, or steel tape, using plumb lines if necessary at M or at S. Before being used in computations, r must be expressed in the same unit as d. The telescope should be so far from M that its draw tube will permit focussing on M.

Primary Measurements. — To find the requisite closeness in the measurement of r we have to note that r enters as a direct factor in the denominator of $\tan \varphi$. Hence the percentage or fractional error in $\tan 2\varphi$ is proportional to that in r, but with the opposite sign. (See

"Computation Rules," Proposition I, page xii.) The limiting value of $\delta r / r$ will therefore be the same, neglecting sign as $\delta d / d$. Therefore

$$\delta r / r = 0.00\,030, \quad \delta r = 0.3 \text{ mm.}$$

Hence the requisite closeness in r is about 0.3 mm.; that is, r must be measured and must be constant to about 0.3 mm. See remark in next paragraph.

SECONDARY MEASUREMENTS. — In secondary work r is not measured, but must remain constant, and suitable means must be provided to see that it is so, within the same closeness of 0.3 mm. This requires much more attention than is usually given to this point, especially when the mirror hangs on a long suspension so that change of level of the instrument may easily change r by 1 or 2 mm.

A special device is almost a necessity in careful work, to facilitate the test for constancy in the distance $O M$. In many instruments, especially those having a long suspending fiber, a slight difference of level alone produces considerable displacements of the mirror towards or from the scale or laterally, — all equally objectionable. The direct application of XII and XIII for the elimination of these displacements on each day of use of the apparatus would be very laborious. This fact together with inadvertence frequently leads to the assumption that the distance, once adjusted, remains sufficiently constant. The foregoing figures as to the requisite closeness show the danger in such neglect. Devices will readily suggest themselves. The following will sometimes answer: Place the points of the leveling screws in positions from which they are not easily displaceable. Level the instrument until the suspension swings as it should. Then make upon the instrument, near the lower end of the suspending fiber, say on opposite sides of the suspension tube, reference marks so located that the fiber lies in the line of sight between them. Similarly locate another line of sight, nearly at right angles to the first and also passing through the fiber. For convenience, one of these lines should be parallel to a pair of the leveling screws, and also to the scale. At each time of use, adjust the screws to bring the fiber into both of these lines of sight. Measure $O M$ once for all, and at the same time measure between two points chosen for convenience, one on the base of the instrument, the other on the scale. It is then necessary from day to day merely to bring the fiber into the reference lines by turning the

leveling screws, and then to measure the distance between the chosen points, for which purpose a rod cut to the right length is convenient.

XV. Estimation of Tenths in Reading.

Fractions of a division must be read, and this can be done only by estimation by the eye. With a little practice, so much facility in the estimation of tenths of a division is attainable that the error need never exceed one-twentieth of a division, with an average error of half this amount, or 0.025 division.

Primary Measurements. — The limit of attainable accuracy being fixed at $\pm d = 0.025$ division, and the desired fractional accuracy being $\pm \delta d / d = 0.00030$, the minimum admissible value of d will be $d = 0.025 / 0.00030 = 83$, or in round numbers $d = 100$ divisions, whatever the size of the division. Smaller deflections than 100 must not be used in the most careful work, and preferably not less than, say, 200 divisions, whatever the scale distance.

Secondary Measurements. — Same as in primary.

XVI. To Find φ, Tan φ, Sin φ, etc., from d and r.

In the application of the telescope and scale method it is desired to find from the observed values of d and r the corresponding values of φ, tan φ, sin φ, etc., according to circumstances.

For the most accurate work, especially in primary measurements, the best way is to compute tan $2\varphi = (d / r)$ or log tan 2φ, thence by tables of natural or log tangents to find 2φ. This, of course, gives φ at once, and tan φ, sin φ, and other functions can then be found from tables. In some cases, notably in secondary work, it is, however, more convenient to have an expression for φ, tan φ, or other function, directly in terms of d and r. Such expressions generally take the form of a series. Several may be found in Czermak. The expression for tan φ is the one in frequent use, and therefore this alone will be given here. It is derived from a development of tan φ in series with ascending powers of tan 2φ, viz.:

$$\tan \varphi = \frac{1}{2} \tan 2\varphi \left[1 - \frac{1}{4} (\tan 2\varphi)^2 + \frac{1}{8} (\tan 2\varphi)^4 - \ldots . \right].$$

Substituting d / r for tan 2φ gives

$$\tan \varphi = \frac{1}{2} \cdot \frac{d}{r} \left[1 - \frac{1}{4} \left(\frac{d}{r}\right)^2 + \frac{1}{8} \left(\frac{d}{r}\right)^4 - \ldots . \right].$$

This could be used directly, but may be modified into a more convenient form. Multiplying the terms of the parenthesis by d gives

$$\tan q = \frac{1}{2r}\left[d - \frac{1}{4}\cdot\frac{d^3}{r^2} + \frac{1}{8}\cdot\frac{d^5}{r^4} - \ldots\right].$$

Then the quantity

$$-\frac{1}{4}\cdot\frac{d^3}{r^2} + \frac{1}{8}\cdot\frac{d^5}{r^4} - \ldots$$

being very small may be conveniently treated as a correction δ to be applied to d. So we may write

$$\tan q = \frac{1}{2r}[d + \delta].$$

The procedure would be to compute beforehand a table of values of this correction for the given value of r and for successive values of d (= 200, 250, 300, . . . 500) at sufficiently short intervals over the desired range. Then for a subsequent observed value of d', the corresponding value of δ' would be taken by interpolation in a table (or upon a plot), and applied to d'. This corrected value of d' would then be multiplied by $1 / 2r$ to obtain $\tan q$. As the correction is small, only an approximate value of r is required in computing it, and slight changes in r during the progress of work would not vitiate the table, although they must be allowed for in the term $1 / 2r$.

In secondary work any of the above methods of finding q, $\tan q$, etc., may be used, but for use in connection with tangent instruments the application of the series expression is especially advantageous. This will be illustrated by the tangent galvanometer. For this instrument the current C is related to the steady, angular deflection q which it produces, by the expression

$$C = k \tan q,$$

where k is a numerical factor which is constant so long as the magnetic conditions and dimensions of the instrument are constant. Substituting the above expression for $\tan q$ gives

$$C = \frac{k}{2r}[d + \delta]$$

or, as r is a constant throughout the work, we may write

$$C = K[d + \delta]$$

where K is a numerical constant $= k / 2r$ and

$$\delta = -\frac{1}{4} \cdot \frac{d^3}{r^2} + \frac{1}{8} \cdot \frac{d^5}{r^4} - \ldots .$$

But in the employment of such a secondary instrument it is calibrated, that is, its constant is found, by sending through it some known current C' and observing the steady deflection d'. Then denoting by δ' the known value of the correction δ computed for this deflection d', we have

$$K = \frac{C'}{d' + \delta'}.$$

This gives us the numerical value of K so that C can be at once computed for any subsequently observed values of d by the expression

$$C = K\,[d + \delta],$$

the proper value of δ being taken from a table or plot as above described. The correction δ will be a quantity to be subtracted from d, and therefore expressed in the same unit, *e. g.*, millimeters. The table must be carried out to the next place of significant figures beyond the last place obtained in reading the scale, *e. g.*, to hundredths of mm. if readings are taken to tenths of mm. It must also be computed for sufficiently short intervals to enable interpolation to be made with corresponding closeness, *i. e.*, to 0.01 mm. or 0.02 mm. in the above case. And if a plot is used it must be on a sufficient scale and with sufficiently frequent points to enable this same closeness to be obtained. Unless many readings of d are to be reduced, it is as well to compute δ for each observation as to make a table. Extensive tables of δ for graded values of r and d are given in Czermak. The number of terms to be retained in the series in computing δ must be determined by computing for the largest value of d the value of the successive terms until one is reached which is negligible, *e. g.*, less than about 0.01 mm. in the above case.

If the instrument is used always with deflections near the value d', observed in calibrating, it is evident that the values of δ both in obtaining K and subsequent measurements will be nearly the same. Hence if δ were omitted in computing K, and also in all subsequent computations of C, the errors thus introduced would nearly offset one another. To see how wide a range of deflections could thus be used

without introducing into C a resultant error exceeding the limit which we have been using in the preceding discussion, we have merely to find the two deflections d_1 and d_2 at any desired part of the scale for which $(\delta_2 - \delta_1) / d_1 = 0.00\ 030$. For a rather extreme case of $d_1 = 400$ mm., $r = 1000$ mm.,

$$\delta_2 - \delta_1 = 0.12 \text{ mm.}$$

The value of d_2 for which δ_2 would be $\delta_{400} + 0.12$ could be found from tables, but in default of these we may compute as follows, neglecting the second term in δ,

$$\frac{1}{4} \cdot \frac{d_2^3}{r^2} - \frac{1}{4} \cdot \frac{d_1^3}{r^2} = 0.12$$

$$\therefore\ d_2^3 - d_1^3 = 0.48 \cdot 10^6$$

$$\therefore\ d_2^3 = 400^3 + 0.48 \cdot 10^6$$

$$= 64 \cdot 48 \cdot 10^6$$

$$\therefore\ d_2 = 401.0 \text{ mm.}$$

Thus the correction could be neglected in this case only over a range of 1 mm. Even with large values of r and smaller ones of d the correction becomes negligible over only a centimeter more or less, and thus is practically never negligible in 0.1 per cent. work.

Selection of Apparatus.

Rigidity of construction is a prime requisite. It is often impaired by unnecessary and weak adjustments or poor clamping devices. The scale may be, and generally is, carried on the same support as the telescope. But an entirely independent support is to be preferred, as facilitating adjustment and promoting stability. This will be the more obvious if it be remembered that the telescope need not be exactly over the center of the scale or in any determinate position relatively to it, although an approximately central position is usually more convenient and is presupposed in the preceding discussion of some of the corrections. A convenient method of supporting the scale would be by a bracket at each end clamped to the table by screws passing through slots in the bracket. This would permit the needed forward and back motion of the ends separately. Long and wide slots in the vertical arms of the brackets would afford the necessary vertical and endwise

range of adjustment. With this arrangement the scale can be placed in final adjustment and clamped in position without employing the telescope, and the latter may be separately put in place with the minimum of trouble, and with no chance of disturbance of the scale.

The telescope must have rotation about both vertical and horizontal axes. It should be provided with a vertical adjustment through a range of six inches or more, preferably by a round rod, which gives at the same time the needed vertical axis. The base must be provided with screw holes for immovable attachment to the table. The draw tube of the telescope must be thoroughly firm, so that a moderate lateral pressure on the eye-piece shall produce no permanent shifting of the cross-wires over the scale. The tube must be long enough to focus on objects at a distance of somewhat less than one meter. The definition of the telescope should be tested as carefully as desired, but will usually be sufficiently good if a clear image of the scale is given under fair illumination at the usual distance.

The definition of the mirror on any instrument to be used with this method must be tested, for example, by the quality of the image of the scale when viewed through the telescope at the usual distance, but with no interposed cover-glass. The effect of the cover-glass on the definition must be carefully tested by alternately inserting and withdrawing it, noting the change in sharpness of the image of the scale. The best of thin plate glass is barely good enough, and selection from samples is often necessary, especially to secure sufficiently parallel surfaces. There is little advantage in having the diameter of the objective of the telescope more than double that of the mirror to be used; an inch to an inch and a half is usually abundant, and three-quarters of an inch is often enough. A magnifying power of twelve to fifteen diameters is desirable but is not common. The relation between size of scale-division, magnifying power, and distance will be briefly considered.

Let $OM = r$, $u =$ magnifying power, and $s_0 =$ the best size of division upon which to estimate tenths of a division by the unaided eye; this is about 1 mm. Then s_0 / v is, nearly enough, the angle in radians subtended by one scale division when seen directly at the distance v of most distinct vision. If at any other distance, as at $TM + MO$, reckoning TM from the eye-piece, the angle becomes $s_0 (TM + MO)$. This is the distance in the telescopic method. In order then that the telescope shall compensate for this removal, that

is, in order that it shall make the arrangement as sensitive as a direct reading at the distance of most distinct vision (which is the condition attainable by the use of a spot of light proceeding from T and reflected from M to the scale at A), the telescope must have a sufficient magnifying power. This must be, for objects at a distance $TM + MO$, such that $s_0 / (TM + MO) = s_0 / v$, or $u = (TM + MO) / v$. For the case where $TM = MO = r = 1000.$ mm., and $v = 250.$ mm., this yields $u = 8$. Telescopes are often furnished for this use with smaller values of u than that just deduced, hence for ordinary use with low power instruments, the millimeter is about the proper size of scale division, but this is by no means the case with higher powers. The best value of s for any fixed value of u would be such that $us / (TM + MO) = s_3 / v$, or $s / s_0 = (TM + MO) / uv$. For a very good glass $u = 15$, so that with the values as before, the scale should be in half millimeters if used at the distance of $r = 1$ meter; since $s / s_0 = 2000 / (15 \times 25) = 0.53$.

With the mirror and spot of light method, the percentage or fractional precision with which a given angle can be measured with the scale at a distance MO is proportional to that distance; that is, the precision for MO is to that for a distance unity as $MO : 1$. With a telescope of magnifying power u the gain is further increased by the telescope in the ratio S / s, where S is the length of division actually employed, and s is the best length computed as just indicated, with the limitation, of course, that S is greater than s. Thus by the employment of unduly long divisions, the advantage derivable from high magnifying power may be sacrificed and most of the advantage over a much cheaper instrument rendered idle. Of course the superiority of the telescopic method over the spot of light does not lie wholly in the magnification, but partly in better definition, and in the avoidance of the necessity for screening. On the other hand, the latter method has the merit of simplicity and cheapness, as well as of facility of reading where there is much jarring and high accuracy is not demanded.

As to materials for the scale, a white metal surface with fine black rulings would be best, and is almost indispensable in accurate work, but is expensive and not generally, if at all, offered by makers. White porcelain or glass with fine black rulings is the next choice. Paper on wood, or celluloid, is not to be relied upon in careful work, and must be thoroughly tested for uniformity.

The warping of wooden scales may introduce serious error (cf. III).

The numbering usually extends from zero at the middle towards each end, but for many purposes a continuous numbering from one end is more convenient (cf. XIII). By the use of a circular scale with its center of curvature at *M*, the readings become directly proportional to twice the angle of deflection, and the focus is equally good throughout the length.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
Boston, Mass., May, 1898.

RESULTS OF TESTS MADE IN THE ENGINEERING LABORATORIES.

X.

Received July 9, 1898.

STEAM.

DESCRIPTION AND RESULTS OF A 45-HOUR TEST MADE ON THE ENGINES, BOILERS, GENERATORS, AND AIR PUMPS AT THE HARVARD SQUARE POWER STATION OF THE BOSTON ELEVATED RAILROAD.

THIS test was made by students of the senior class under the direction of the instructors of the Engineering Laboratories at the Massachusetts Institute of Technology.

The test was divided into four watches of $11\frac{1}{4}$ hours, twenty-four students working at each watch. During the test 2,000 indicator cards were taken, and about 10,000 observations recorded. In the accompanying tables such summaries of these observations are given as were needed in the calculations of the results.

The power station has at present six Babcock & Wilcox boilers set in three batteries. The boilers all have extension furnaces. The gases are discharged into a common flue at the back of the boilers and pass from this through a Green economizer to the stack. The boilers are fed by power plunger pumps located in the basement of the engine room. These pumps are driven from a countershaft run by an electric motor supplied with current from the station. There are three main engines, $28'' - 56'' \times 5'$, of the cross-compound jet-condensing type, made by E. P. Allis & Company, of Milwaukee.

There is an independent condenser and air pump engine for each main engine. The main engines have Corliss valves with two wrist

plates and eccentrics for each cylinder, thus making it possible to cut off later than half stroke on each cylinder.

The air pump engines are controlled by throttling governors, the valves being cylindrical D valves oscillating in cylindrical chests at right angles to the bore of the cylinder.

The main engines are set sufficiently high on their foundations to allow all the steam and exhaust piping to be placed below the engine room floor. The arrangement of this piping is similar to that at the Sullivan Square station at Charlestown, Mass. (See *Technology Quarterly*, Vol. IX, No. 4.) A separator is placed just before the throttle; the exhaust from the high-pressure cylinder passes into a vertical receiver from which the low-pressure cylinder takes its steam. The exhaust from the low-pressure cylinder passes to the condenser or outboard.

On each exhaust pipe just before the condenser there is a feed-water heater. The feed-water, coming through a 6″ pipe from the city main, passes through these heaters on the exhaust pipes, then through the economizer to the suction side of the power pumps, and from the pumps directly to the boilers. By this arrangement the economizer is under only such pressure as there may be in the city main.

The drip from the separators on steam mains is trapped into the suction pipe of the pumps. The drip from the receivers, between the cylinders, contains considerable oil and is thrown away.

The air pumps draw the condensing water from the Charles River through a 20″ pipe about 300 feet long. About 100 feet from the station the suction and discharge pipes of the pumps are connected by cross-overs and valves, so that the outer end of either pipe may be used as suction or discharge. At this place there was also a second discharge, which could be used in case of accident to one of the other pipes.

The generators are direct connected, of the M. P. 12–1200–80 type, built by the General Electric Company. The armature is located upon the engine shaft. The nominal output of each machine is 2,180 amperes at 550 volts. Each generator is provided with a separate panel upon the switchboard gallery, on which is located a shunt-ammeter, wattmeter, circuit-breaker, switches, a rheostat, etc.

Upon a separate panel is a station shunt-ammeter that registers the total output of the station. A station voltmeter is also provided

upon a swinging bracket, and connections arranged so that it can be connected to either generator.

The customary complement of feeder switches, ammeters, circuit breakers, etc., are provided and mounted upon slate panels.

Arrangement and Changes Made for the Test.

Lazy-tongs were attached to each crosshead to give the proper reduction of piston motion for 2″ diameter drums on the indicators. Indicators were attached to each end of the cylinders of main engines No. 2 and No. 3, the ones used during the test. Air-pump engines No. 2 and No. 3 were each piped up with three-way cocks. The motion for the drums of these indicators was taken from a pin screwed into the end of the shaft. Although this does not give a correct motion, it was considered to be sufficiently accurate, as the horse power of these engines is not considered in the subsequent calculation.

The drips from the receivers were caught in a tank on scales and weighed. The drip pipes from the separators were each disconnected from the traps and each connected with a steel reservoir about 16″ in diameter and 48″ in height, having on the side a gauge glass and scale giving the capacity between different levels. From time to time the levels were blown down by opening valves in the bottom, the discharge going into the suction pipe of the power pumps.

The feed-water pipe was broken at the pumps on the suction side, and the suction pipe changed so as to draw from three large barrels placed alongside the pumps. A check valve opening towards the pumps was placed in the suction pipe close to the barrels. The discharge from the separators was connected between this check valve and the pumps.

The regular suction pipe was carried up to the floor above and supplied four large barrels used for weighing. These barrels discharged into the three below connected with the suction of the pumps. The four weighing barrels were supplied from the city main with water which had passed through the heaters on the exhaust pipe. Ordinarily the water went through the economizer as well, before entering the pumps. During the test the pumps forced the water through the economizer into the boiler. It will be noticed that the temperature of the feed water entering the economizer is less, by a few degrees, than when leaving the heater on the exhaust pipe. This loss is due to

the exposure to the air while weighing. Wooden scales were attached to the water glasses on the boilers.

Calorimeters were placed on the main steam pipe from each battery and near the throttle of No. 2 or No. 3 engine.

Holes for flue thermometers and draught gauges were made at each end of the economizer, and connections for sampling flue gases were made at the entrance end.

The coal scales were tested. The different gauges at the gauge boards in the engine room were tested and the corrections noted. With one exception, all connections on the feed water piping, where there was a possibility of leakage, were broken and blanked.

The place referred to as not being blanked was where an auxiliary steam pump connected with the line. There were two Chapman valves here, one either side of the connection. A pet-cock was tapped into the pipe between these valves and left open, so that any leakage would be noticed.

A weir box 10 feet long, and having a weir 3 feet wide, without end contractions, was placed at the end of the auxiliary discharge pipe for condensing water. The pass valves were changed so as to send the water over the weir. The height of water on the weir was measured by a hook gauge reading to $\frac{1}{1000}$ of a foot. The water entering the weir box was quieted by straining through about 3 feet of brush and a screen of clapboards.

The other observations taken during the test may be noted by referring to the list of stations.

STATIONS.

(1) Electrical readings.

(2) Electrical readings.

(3) Coal, temperature, boiler room, barometer.

(4) Gas analysis every two hours. Draught and temperature of flue at each end of economizer every half-hour, also temperature of feed at top of boilers every half-hour.

(5) Gas analysis every two hours. Temperature of steam in main of batteries No. 1 and No. 3 every fifteen minutes; also calorimeters on boilers No. 1 and No. 3.

(6) Water barrel No. 6.

(7) Water barrel No. 7.

(8) Water barrel No. 8.

(9) Water barrel No. 9.

(10) Indicator, planimeter, length and per cent. cut-off, No. 2 H. H.

(11) Indicator, planimeter, length and per cent. cut-off, No. 2 H. C.

(12) Indicator, planimeter and length, No. 2 L. H.

(13) Indicator, planimeter and length, No. 2 L. C.

(14) Indicator, planimeter, length and per cent. cut-off, No. 3 H. H.

(15) Indicator, planimeter, length and per cent. cut-off, No. 3 H. C.

(16) Indicator, planimeter and length, No. 3 L. H.

(17) Indicator, planimeter and length, No. 3 L. C.

(18) Indicator, planimeter and length. Air pump No. 2 engine.

(19) Indicator, planimeter and length. Air pump No. 3 engine.

(20) Time, gong (every fifteen minutes) thus: . . . twenty seconds . . ten seconds . Counter engine No. 2, one-half minute after gong. Counter engine No. 3, one minute after gong. Counter air pump No. 3, one and one-half minutes after gong. Counter air pump No. 2, two minutes after gong. Pressure at gauge boards No. 2 and No. 3 engines. Hook gauge and temperature at weir every half-hour.

(21) Separator drip cans, No. 2 and No. 3 engines.

(22) Temperature feed at city main, leaving heater No. 2 engine, leaving heater No. 3 engine, entering the economizer, and leaving the economizer; also calorimeter near throttle No. 3 engine.

(23) Weight of receiver drips, No. 2 and No. 3 engines, also temperature hot and cold condensing water from air pumps, No. 2 and No. 3 engines.

(24) General log.

The circuit to the motor running the power pumps was broken, and an ammeter inserted, upon which readings were taken and the power required for feeding the boilers obtained.

The shunt field circuit of first one and then the other of the generators was connected through an ammeter and the current required for the fields was thus obtained. A standard voltmeter was connected to the circuit, and check readings obtained upon the station voltmeter.

During the night, in order to obtain a load for one engine after the load on the outside lines became too light, a large water rheostat was provided and connected to No. 3 generator on the first night, and to No. 2 generator on the second night, and a steady load of about the nominal capacity of the machine carried, the other engine being shut down.

A standard shunt and a portable ammeter were connected in this circuit and used when the rheostat was in circuit. Electrical readings upon all instruments were taken every five minutes, every third reading being taken upon the stroke of a gong.

The test on the boilers was continuous for 45 hours. The condition of the fires was noted at the beginning of the run, and the fires were brought to the same condition at the end.

If all the possible errors of the test on the boilers are assumed to be cumulative, the maximum error possible is 1.5 per cent. The tests on the engines and generators were divided up into five parts.

From 1-15 P.M. May 10, to 11-30 P.M. Engines No. 2 and No. 3 on regular station load.

From 12-45 A.M. May 11, to 5-45. Engine No. 3 alone on water rheostat with constant load.

From 6-30 A.M. May 11, to 11-30 P.M. Engine No. 2 and No. 3 on station load.

From 12-45 A.M. May 12, to 5-45. Engine No. 2 alone with constant load by water rheostat.

From 6-30 A.M. May 12, to 10-15 A.M. Engine No. 2 and No. 3 on station load.

The air pumps exhaust into the receivers between the high and low cylinders. During the first run No. 2 air pump exhausted into the condenser till 2-45, when the exhaust was turned into the receiver.

During the last run the exhaust of No. 3 pump was turned into the condenser at 8 A.M.

At the end of each run the levels in the boilers were noted, and if different from those at the beginning of the run, corrections were made for the difference.

The B. T. U. per horse power per minute of main engines was calculated by multiplying the steam per H. P. per minute by the heat in a pound of steam, of condition as determined by a calorimeter at the throttle, above the heat of the feed water leaving the heater on the exhaust pipe.

The coal per indicated horse power per hour of main engines was calculated by dividing the B. T. U. per horse power per hour by the B. T. U. taken up from a pound of dry coal. The probable error of work depending on the indicator may be assumed to be about 2 per cent.

SUMMARY OF DATA AND RESULTS OF A 45-HOUR TEST AT HARVARD SQUARE POWER STATION OF THE BOSTON ELEVATED RAILROAD.

Engine Sizes.

Main engines No. 2 and No. 3.
Diameter high, 28″. Diameter rod, 5½″.
Diameter low, 56″. Diameter rod, 6½″
Stroke, 5 feet.

Air Pump. Sizes No. 2 and No. 3.

Diameter steam cylinder	16″
Diameter piston rod	2 11/16″
Stroke	12″
Diameter bucket	36″

Boiler Sizes.

(4) B. and W. Boilers.

Heating surface (outside) one boiler	5340 sq. ft.
Grate surface, one boiler	84. sq. ft.
Total heating surface	21,360 sq. ft.
Total grate surface	336. sq. ft.

Weir.

Weir 3 feet long without end contractions.
Crest of weir above floor of pit was 1.6 feet.
Calculate by formula of Fteley & Stearns.
$Q = 3.31\ L\ H^{\frac{3}{2}} + .007\ L$ first approximation.
Correct for velocity of approach, using for H (the height by hook gauge $+ \frac{1.5 v^2}{2g}$).
v = velocity of approach.
L = length of weir.
Q = cubic feet per second.
g = 32.2 feet.

Boiler Test.

Duration of test	45 hours.
Kind of coal	New River.
Boilers B. & W. with extended furnaces (2) batteries of two.	
Total heating surface of the four boilers	21,360 sq. ft.
Total grate surface of the four boilers	336 sq. ft.
Average absolute boiler pressure at boilers	168.4 lbs.
Average quality of steam (at boilers) from both batteries (dry steam = 1),	.989
Total weight of coal as fired from barrows	159,116 lbs.
Total weight of dry coal (1 per cent. of moisture)	157,525 lbs.
Total weight of ashes and clinkers	9,342 lbs.
Total combustible burned	148,183 lbs.
Dry coal burned per square foot of grate per hour	10.42 lbs.
Total water weighed in barrels	1,490,621 lbs.
Total returns from separator drips	7,568 lbs.
Total feed water supplied to boilers	1,498,189 lbs.
Average temperature of feed water entering boilers	209.1° F.
Average temperature of feed water entering economizer	52.7° C. 126.86° F.
Average temperature of feed water leaving economizer	98.5° C. 209.3° F.
Equivalent evap. from and at 212° per pound dry coal (boilers and economizer)	10.68 lbs.
Total B. T. U. taken up by boiler and economizer per pound of dry coal	10,311 B. T. U.
Heat taken up by water in economizer per pound of dry coal	786.4 B. T. U.
Heat gained in economizer in per cent. total heat acquired	7.62
Ash and clinkers in per cent. of total dry coal	5.9

ENGINE TESTS.

Time of run	MAY 10, 1-15 P.M. TO 11-30 P.M.			
Engine number	No. 2.	Pump No. 2.	No. 3.	Pump No. 3.
Revolutions per minute during run	71,974	54,254	72,252	51,531
Average per cent. of cut-off on high pressure cylinder	23.6	.....	18.4	.....
Mean effective pressure on H. H.	58.9	32.1	47.0	37.7
Mean effective pressure on H. C.	54.9	10.4	43.8	4.38
Mean effective pressure on L. H.	8.73	.....	10.9	.....
Mean effective pressure on L. C.	12.0	.....	9.82	.....
Horse power by indicator on H. H.	395.51	16.61	316.82	11.83
Horse power by indicator on H. C.	354.42	3.34	283.85	1.34
Horse power by indicator on L. H.	234.48	.....	293.93	.....
Horse power by indicator on L. C.	317.97	.....	261.22	.....
I. H. P. of Each Engine	1302.4	13.95	1155.8	13.17
Total I. H. P. Both Engines	.....	2458.2		.....
Total water supplied to boilers during run	.....	366,824		.....
Total steam used by three calorimeters during run	.....	12,761		.....
Steam supplied to engines and pumps	.....	354,063		.....
Steam per Hour per I. H. P. of Main Engines	.....	14.05		.....
Temperature of feed water, Entering heater on exhaust °F.	50.8	.....	50.8	.....
Temperature of feed water, Leaving heater on exhaust °F.	132.1	.....	140.2	.....
B. T. U. per I. H. P. of Main Engines per Minute	.....	253.		.....
Coal per I. H. P. of Main Engines per Hour	.....	1.47		.....
Temperature of condensing water, Cold °C.	.....	13.1	.....	12.8
Temperature of condensing water, Hot °C.	.....	29.2	.....	31.4
Weight of drip trapped from working side of receivers	.....	20,600		.....
Quality of steam at throttle. (Dry steam = 1)	.....	.....	.989	.....
Pressure at throttle by gauge at board	149.0	.....	148.4	.....
Pressure in receiver	5.0	.....	9.4	.....
Vacuum in condenser. (Lbs.)	13.2	.....	12.3	.....
Barometer. (Inches of mercury)	.....	30″		.....
Draught Pressures and Flue Temperatures.				
Average temperature of gases, Entering economizer °C.	.....	236.2		.....
Average temperature of gases, Leaving economizer °C.	.....	123.3		.....
Temperature of gases leaving economizer, Maximum °C.	.....	129.0		.....
Temperature of gases leaving economizer, Minimum °C.	.....	111.0		.....
Av. draught pressure in ins. of water, Entrance to economizer,	.....	.5514		.....
Av. draught pressure in ins. of water, Leaving economizer	.....	.4026		.....
Draught leaving economizer, Maximum	.....	.844		.....
Draught leaving economizer, Minimum	.....	.266		.....
From Weir Measurements.				
Average height of water on weir. (Feet.)	.....	.533		.....
Average temperature of water at weir °C.	.....	31.7		.....
Pounds of water over weir per hour	.....	888,459		.....
Pounds of steam condensed per hour	.....	34,542		.....
Condensing Water per Pound of Steam	.....	24.7		.....
Condensing Water per Hour per I. H. P. of Main Engines	.....	344.		.....
From Electrical Measurements.				
Electrical H. P. Output Each Engine	1156	.....	1051	.....
Electrical H. P. Output Both Engines	.....	2207		.....
Efficiency per cent. Ratio of electrical to steam H. P.	.....	89.8		.....
Coal per Electrical H. P. Ouput. (Per hour.)	.....	1.64		.....
Electrical H. P. to run feed pumps	.....	15.0		.....

ENGINE TESTS.

	May 11, 12-45 A.M. to 5-45 PM.		May 11, 6-30 A.M. to 11-30 P.M.			
Time of run						
Engine number	No. 3.	Pump No. 3.	No. 2.	Pump No. 2.	No. 3.	Pump No. 3.
Revolutions per minute during run	71,967	50,420	70,828	55,123	72,075	55,659
Average per cent. of cut-off on high pressure cylinder	33.1	.....	23.43	....	20.60	....
Mean effective pressure on H. H.	67.8	42.4	61.7	31.0	49.4	36.0
Mean effective pressure on H. C.	68.4	3.27	58.8	11.0	47.7	3.3
Mean effective pressure on L. H.	13.2	.....	9.36	....	11.8	....
Mean effective pressure on L. C.	12.6	.....	10.40	....	10.5	....
Horse power by indicator on H. H.	455.21	13.02	407.71	10.41	332.18	12.25
Horse power by indicator on H. C.	441.53	.98	373.55	3.59	308.37	1.09
Horse power by indicator on L. H.	354.51	.....	247.40	....	317.38	....
Horse power by indicator on L. C.	333.84	.....	271.19	....	278.62	....
I. H. P. of Each Engine	1585	14.0	1299.8	14.00	1236.6	13.34
Total I. H. P. Both Engines	.....		....	2536.4		....
Total water supplied to boilers during run	120,575		....	631,354		....
Total steam used by three calorimeters during run	6,225		....	21,165		....
Steam supplied to engines and pumps	114,350		....	610,189		....
Steam per Hour per I. H. P. of Main Engines	14.43		....	14.15		....
Temperature of feed water, Entering heater on exhaust °F.	50.1	....	50.9	....	50.9	....
Temperature of feed water, Leaving heater on exhaust' °F.	162.1	....	133.3	....	142.0	....
B. T. U. per I. H. P. of Main Engine per Minute	254		....	254		....
Coal per I. H. P. of Main Engines per Hour	1.47		....	1.48		....
Temperature of condensing water, Cold °C.	....	10.6	....	13.3	....	11.8
Temperature of condensing water, Hot °C.	....	30.0	....	30.8	....	32.9
Weight of drip trapped from working side of receivers	7580		....	42,360		....
Quality of steam at throttle. (Dry steam = 1)	.990	....	....	....	.990	....
Pressure at throttle by gauge at board	150.5	....	149.0	....	148.2	....
Pressure in receiver	10.4	....	2.8	....	9.0	....
Vacuum in condenser. (Lbs.)	10.2	....	13.0	....	12.2	....
Barometer. (Inches of mercury.)	30		....	30		....
Draught Pressures and Flue Temperatures.						
Average temperature of gases, Entering economizer °C.	225.8		....	240.6		....
Average temperature of gases, Leaving economizer °C.	127.6		....	126.8		....
Temperature of gases leaving economizer, Maximum °C.	133.0		....	134.0		....
Temperature of gases leaving economizer, Minimum °C.	117.0		....	117.0		....
Av. draught pressure in ins. of water, Entrance to economizer,	.4175		....	.6723		....
Av. draught pressure in ins. of water, Leaving economizer	.2902		....	.3632		....
Draught leaving economizer, Maximum	.824		....	.912		....
Draught leaving economizer, Minimum	.308		....	.288		....
From Weir Measurements.						
Average height of water on weir. (Feet.)	.459		....	.535		....
Average temperature of water at weir °C.	32.0		....	31.5		....
Pounds of water over weir per hour	708,851		....	893,429		....
Pounds of steam condensed per hour	22,870		....	35,893		....
Condensing Water per Pound of Steam	30.0		....	23.9		....
Condensing Water per Hour per I. H. P. of Main Engines	432.		....	338.		....
From Electrical Measurements.						
Electrical H. P. Output Each Engine	1409	....	1194	....	1092	....
Electrical H. P. Output Both Engines	1409	....	....	2286		....
Efficiency per cent. Ratio of electrical to steam H. P.	88.8	....	....	90.1		....
Coal per Electrical H. P. Output. (Per hour.)	1.66	....	....	1.64		....
Electrical H. P. to run feed pumps	13.4	....	....	15.3		....

ENGINE TESTS.

	May 12, 12-45 A.M. to 5-45 A.M.		May 12, 6-30 A.M. to 10-15 A.M.			
Time of run						
Engine number	No. 2.	Pump No. 2.	No. 2.	Pump No. 2.	No. 3.	Pump No. 3.
Revolutions per minute during run	71,703	56,957	71,635	58,982	72,195	58,142
Average per cent. of cut-off on high pressure cylinder	36.1	....	19.0	....	20.2	....
Mean effective pressure on H. H.	81.1	38.4	56.7	19.0	45.1	27.2
Mean effective pressure on H. C.	74.9	13.7	54.6	14.0	47.5	7.76
Mean effective pressure on L. H.	11.7	....	8.20	....	11.0	....
Mean effective pressure on L. C.	11.8	....	8.96	....	9.70	....
Horse power by indicator on H. H.	542.52	13.33	378.03	6.82	303.77	9.64
Horse power by indicator on H. C.	481.71	4.62	350.82	4.89	307.59	2.67
Horse power by indicator on L. H.	313.07	....	219.21	....	296.36	....
Horse power by indicator on L. C.	311.50	....	236.3	....	257.82	....
I. H. P. of Each Engine	1648.8	17.95	1185.3	11 72	1165.5	12.31
Total I. H. P. Both Engines	.....		....	2350 8		....
Total water supplied to boilers during run	130,950		....	133,741		....
Total steam used by three calorimeters during run	6,225		....	4,669		....
Steam supplied to engines and pumps	124,725		....	129,072		....
Steam per Hour per I. H. P. of Main Engines	15.13		....	14 64		....
Temperature of feed water, Entering heater on exhaust °F.	52.0	....	52.0	....	52.0	....
Temperature of feed water, Leaving heater on exhaust °F.	147.4	....	133.5	....	143.0	....
B. T. U. per I. H. P. of Main Engines per Minute	269.		....	263.		....
Coal per I. H. P. of Main Engines per Hour	1.56		....	1.53		....
Temperature of condensing water, Cold °C.	....	11.8	....	13.3	....	13.1
Temperature of condensing water, Hot °C.	....	25.8	....	29.2	....	32.2
Weight of drip trapped from working side of receivers	7170		....	8,830		....
Quality of steam at throttle. (Dry steam = 1)	.989	....	.989	....	....	....
Pressure at throttle by gauge at board	149.9	....	151.3	....	150.4	....
Pressure in receiver	3.7	....	2 1	....	10.1	....
Vacuum in condenser. (Lbs.)	12.3	....	12.9	....	11.8	....
Barometer. (Inches of mercury.)	30		....	30		....
Draught Pressures and Flue Temperatures.						
Average temperature of gases, Entering economizer °C.	217.7		....	236.3		....
Average temperature of gases, Leaving economizer °C.	126.4		....	132.3		....
Temperature of gases leaving economizer, Maximum °C.	129.0		....	139.0		....
Temperature of gases leaving economizer, Minimum °C.	121.0		....	124 0		....
Av. draught pressure in ins. of water, Entrance to economizer,	.6891		....	.4090		....
Av. draught pressure in ins. of water, Leaving economizer	.4316		....	.3095		....
Draught leaving economizer, Maximum	.846		....	.682		....
Draught leaving economizer, Minimum	.298		....	.246		....
From Weir Measurements.						
Average height of water on weir. (Feet.)	.451		....	.454		....
Average temperature of water at weir °C.	30.4		....	36.3		....
Pounds of water over weir per hour	691,108		....	696,585		....
Pounds of steam condensed per hour	24,945		....	34,418		....
Condensing Water per Pound of Steam	26.7		....	19.3		....
Condensing Water per Hour per I. H. P. of Main Engines	404.		....	281.		....
From Electrical Measurements.						
Electrical H. P. Output Both Engine	1499	....	1083	....	1014	....
Electrical H. P. Output Both Engines	1499	....	....	2097		....
Efficieney per cent. Ratio of electrical to steam H. P.	90.9	....	....	89.2		....
Coal per Electrical H. P. Output (Per hour.)	1.72	....	....	1.72		....
Electrical H. P. to run feed pumps	14.7	....	....	14.5		....

TESTS ON THE TRIPLE EXPANSION ENGINE, 9″–16″–24″ × 30″.

During the last six years quite an extended series of tests have been made on this engine to determine the effect of steam jacketing. The results have been published in the *Technology Quarterly*, and also in the *Trans. Am. Society of Mechanical Engineers.*

The following tests form the beginning of a new series, having for objects, first, the gain to be derived by using reheating surface on the receivers only; second, the amount of reheating surface needed for best results. The tests are printed at this time because of the considerable interest taken in this work by designers of steam engines.

Up to the summer of 1897 the engine had double shell receivers made by the E. P. Allis Company, with jacket steam supplied to the space between the two shells. The reheating surface of these receivers was not very effective. During the summer of 1897 these two receivers were taken out and Wainwright reheaters of special design substituted. These new reheaters are so constructed that either one-third, two-thirds, or the entire amount of reheating surface can be used. The entire jacket surface of the heaters was used in the tests of which the results are given below.

A working drawing of the second reheater is shown in Figure 1.

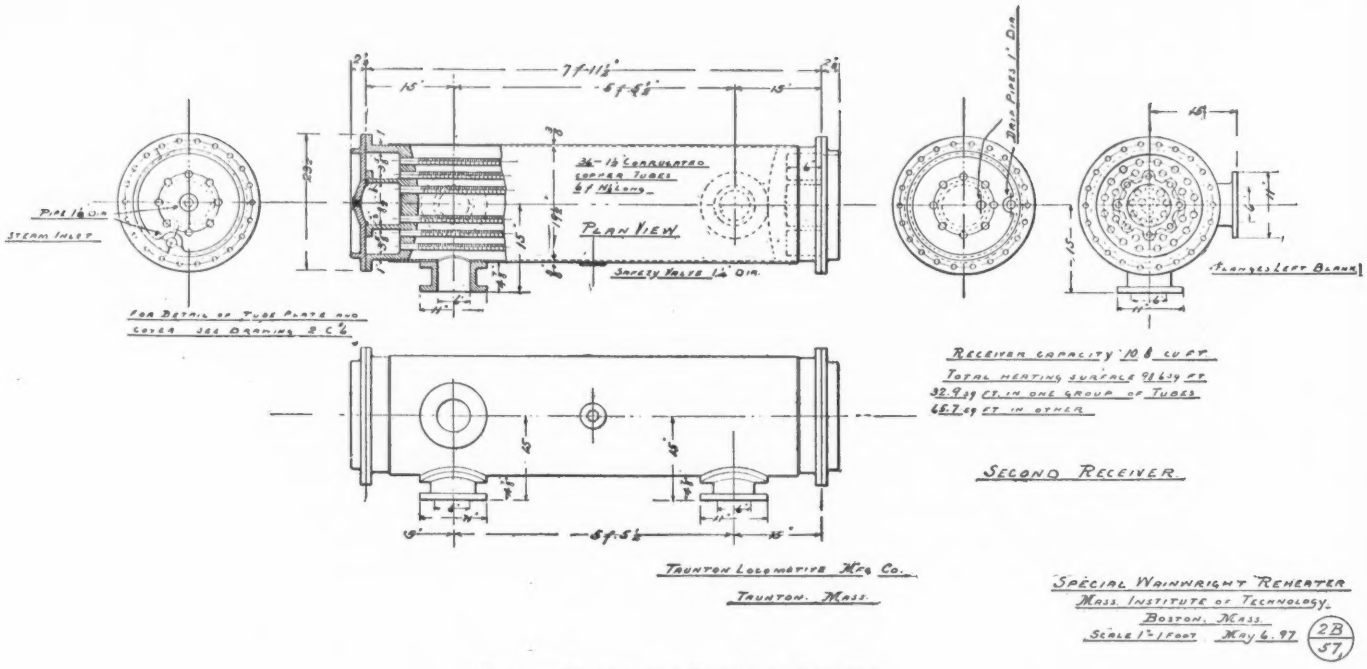

FIG. 1.—WAINWRIGHT REHEATING RECEIVER.

TESTS ON THE TRIPLE EXPANSION ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Date.	Horse power.	Steam per horse power per hour.	Steam through cylinders per hour.	Water by Jackets Per Hour.		Revolutions per minute.	Boiler pressure gauge.	Vacuum in condenser. (Inches of mercury.)	Barometer.
				First receiver.	Second receiver.				
Mar. 8, 1898 . .	88.56	15.95	14.13	....	....	81.82	146.7	26.37	30.61
Mar. 7, 1898 . .	87.52	16.03	14.03	....	....	81.80	147.5	26.05	30.39
Mar. 4, 1898 . .	89.68	16.01	14.36	....	....	81.60	147.0	25.89	30.51
Mar. 29, 1898 . .	103.32	15.53	16.05	....	....	81.20	148.2	25.90	30.16
May 6, 1898 . .	66.45	15.71	9.21	....	123.3	85.54	147 3	25.48	29.98
May 9, 1898 . .	84.93	15.87	11.96	....	152.6	83.25	146.9	23.80	30.16
Mar. 22, 1898 . .	112.38	15.01	15.05	....	183.0	81.40	146.1	25.80	30.17
Apr. 8, 1898 . .	61.53	15.53	8.07	80.2	58.1	84.97	147.3	26.59	30.32
Mar. 14, 1898 . .	74.78	14.90	9.32	108.9	76 9	84.45	146.9	26.16	30.31
Mar. 15, 1898 . .	95.65	14.72	12.21	117.8	70.2	82.43	147.1	25.33	30.44
Apr. 1, 1898 . .	105.88	14.69	13.59	81.9	115.6	81.92	147.7	25.40	30.10
Mar. 28, 1898 . .	107.00	14.54	13.64	90.0	102.8	81.97	146.6	25.72	39.22

Date.	High Pressure Cylinder.									
	Initial pressure.	Per cent. of cut-off.	Pressure at cut-off.	Pressure at release.	Pressure at compression.	Per cent. of steam in the cylinder at cut-off.	Per cent. of steam in the cylinder at release.	M. E. P. crank.	M. E. P. head.	Horse power.
Mar. 8, 1898 . .	142.1	27.0	138.4	37.4	40.6	76.3	84.3	48.7	47.8	36.86
Mar. 7, 1898 . .	142.5	27.3	132.4	37.3	39.3	73.1	82.2	50.1	49.2	37.92
Mar. 4, 1898 . .	142.2	29.2	128.5	36.7	41.4	75.9	82.4	51.9	49.3	38.50
Mar. 29, 1898 . .	144.3	35.5	135.6	48.8	50.8	81.6	88.9	52.1	51.5	39.25
May 6, 1898 . .	142.5	10.0	134.0	16.7	15.1	66.7	88.3	45.0	28.4	29.11
May 9, 1898 . .	139.5	18.9	128.8	27.0	30.0	69.8	83.6	49.9	36.3	33.30
Mar. 22, 1898 . .	143.8	31.1	134.3	42.0	44.3	78.3	86.2	51.0	49.9	38.31
Apr. 8, 1898 . .	145.4	7.8	125.4	12.2	15.9	69.9	83.7	34.0	27.1	24.10
Mar. 14, 1898 . .	146.1	10.0	136.2	17.1	19.8	63.4	84.6	39.2	30.7	27.43
Mar. 15, 1898 . .	146.2	20.5	135.0	29.4	32.3	72.4	85.1	46.3	42.8	34.20
Apr. 1, 1898 . .	144.8	26.6	132.2	38.7	45.6	75.3	89.0	44.6	43.4	33.62
Mar. 28, 1898 . .	145.0	27.8	130.6	37.8	43.4	76.6	87.0	46.9	46.9	35.91

TESTS ON THE TRIPLE EXPANSION ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY. — *Continued.*

Date.	Intermediate Pressure Cylinder.									
	Initial pressure.	Per cent. cut-off.	Pressure at cut-off.	Pressure at release.	Pressure at compression.	Per cent. of steam in the cylinder at cut-off.	Per cent. of steam in the cylinder at release.	M. E. P. crank.	M. E. P. head.	Horse power.
Mar. 8, 1898 . .	36.6	15.5	26.7	— 2.12	+ 0.37	55.1	76.3	11.7	11.7	28.85
Mar. 7, 1898 . .	34.1	17.5	24.1	— 1.93	+ 0.00	56.2	77.4	10.7	11.5	27.49
Mar. 4, 1898 . .	35.3	17.6	26.3	— 1.30	+ 0.75	57.3	78.5	10.8	11.9	27.84
Mar. 29, 1898 . .	43.2	15 9	35.1	+ 0.67	+ 1.51	58.2	79.3	15.6	14.2	36.52
May 6, 1898 . .	13.3	19.2	9 52	— 4.30	+ 3.0	58.1	95.9	4.81	4.50	12.03
May 9, 1898 . .	25.7	17.5	19.5	— 0.80	+ 0.68	56.5	94.4	6.81	7.22	17.65
Mar. 22, 1898 . .	39.4	17.5	32.7	+ 1.39	+ 3.82	60.1	85.2	10.9	12.3	28.54
Apr. 8, 1898 . .	11.3	25.7	8.19	— 4.60	+ 2.99	74.4	Super.	5.08	5.51	13.61
Mar. 14, 1898 . .	14.4	32.1	9.38	— 2.78	+ 1.89	78.9	Super.	6.43	7.19	17.39
Mar. 15, 1898 . .	28.6	24.6	23.0	+ 0.95	+ 2.26	75.2	Super.	9.07	10.6	24.57
Apr. 1, 1898 . .	39.2	18.8	32.1	+ 1.05	+ 3.63	69.4	92.4	12.1	12.5	30.37
Mar. 28, 1898 . .	37.5	20.2	32.4	+ 1.07	+ 2.75	73.0	92.5	12.1	12.2	30.08

Date.	Low Pressure Cylinder.										B. T. U. per H. P. per minute (actual).	B. T. U. per H. P. per minute reduced to 26″ vacuum.
	Initial pressure.	Per cent. of cut-off.	Pressure at cut-off.	Pressure at release.	Pressure at compression.	Per cent. of steam in the cylinder at cut-off.	Per cent. of steam in the cylinder at release.	M. E. P. crank.	M. E. P. head.	Horse power.		
Mar. 8, 1898 . .	—2.6	22.6	—5.2	—11.1	—12.0	44.9	61.3	4.14	3.99	22.85	289.7	288.4
Mar. 7, 1898 . .	—2.3	19.5	—5.4	—11.1	—11.9	40.0	55.3	3.97	3.90	22.11	290.9	288.9
Mar. 4, 1898 . .	—1.3	18.7	—4.4	—10.6	—11.2	41.2	64.4	4.25	4.08	23.34	289.8	286.3
Mar. 29, 1898 . .	—0.7	22.5	—3.0	—10.2	—11.3	46.8	61.7	4.81	5.07	27.55	282.0	280.8
May 6, 1898 . .	—3.6	36.8	—5.7	—10.6	—11.3	72.1	98.8	4.06	4.54	25.31	277.1	272.7
May 9, 1898 . .	+0.7	35.5	—2.9	—10.0	—11.2	74.7	86.0	6.13	5.76	33.98	277.1	261.5
Mar. 22, 1898 . .	+2.3	39.4	—1.6	— 9.1	—11.4	82.9	81.9	7.97	8.32	45.53	265.8	264.4
Apr. 8, 1898 . .	—3.8	32.6	—6.0	—11.1	—12.0	92.1	Super.	3.92	4.24	23.82	268.6	274.4
Mar. 14, 1898 . .	—1.7	33.3	—5.3	—11.0	—12.1	86.7	90.2	5.19	5.14	29.06	260.7	259.9
Mar. 15, 1898 . .	+1.9	33.2	—2.5	—10.3	—11.6	83.7	80.8	6.46	6.57	36.88	257.8	252.0
Apr. 1, 1898 . .	+1.8	35.7	—1.4	— 9.4	—11.5	79.8	85.4	7.20	7.63	41.89	258.6	254.5
Mar. 28, 1898 . .	+1.7	36.0	—1.4	— 9.1	—11.6	86.1	89.6	7.14	7.43	41.01	256.4	254.5

Cranks set 120° apart, the high leading.

Total volume of working side of first reheater, 8.5 cubic feet. Second reheater, 11 cubic feet.

Total reheating surface, first reheater, 80.2 square feet. Second reheater, 102.4 square feet.

Volume in piping and ports between exhaust valves on high, and admission valves on intermediate, exclusive of reheater, 22.3 cubic feet.

Volume in piping and ports between exhaust valves on intermediate, and admission valves on low, exclusive of reheater, 20.4 cubic feet.

Steam used in the jackets was supplied at full boiler pressure.

Jackets on the cylinders were not in use.

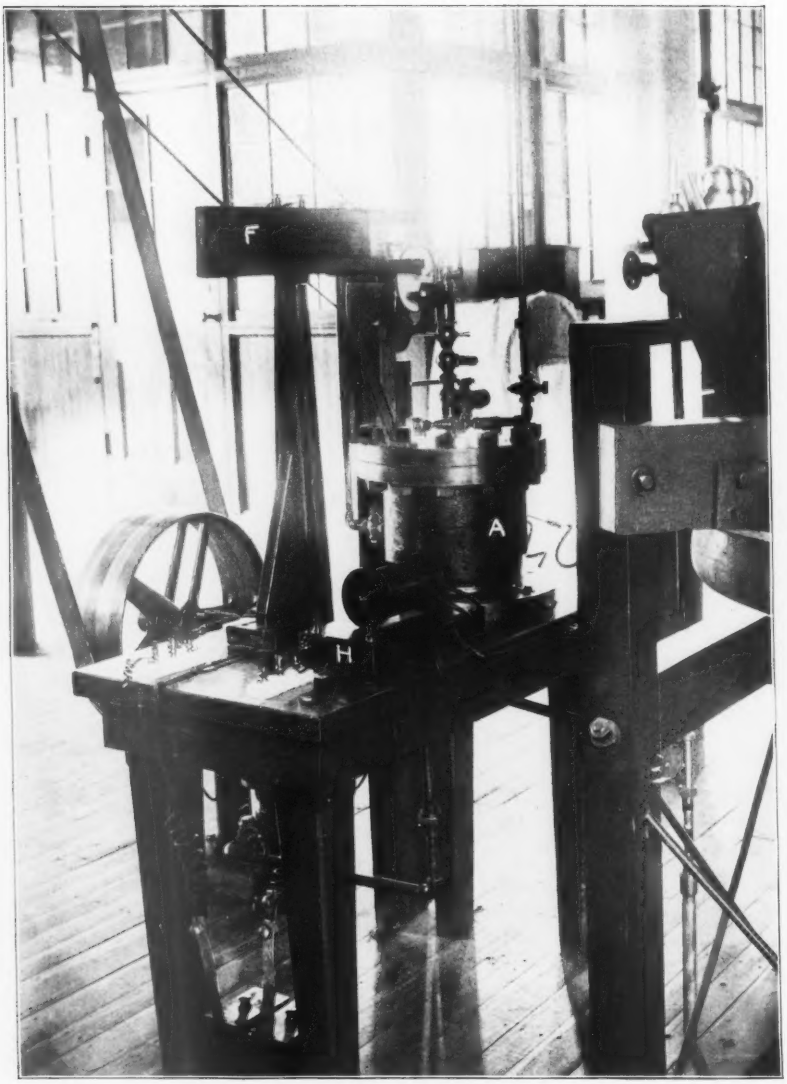

FIG. 2. — APPARATUS FOR TESTING EXPLOSIVE MIXTURES OF ILLUMINATING GAS AND AIR.

Explosive Mixtures of Illuminating Gas and Air.

During the last year a few experiments have been made upon explosive mixtures of illuminating gas and air. These experiments have formed a part of the regular work in the fourth year mechanical engineering laboratory.

The essential parts of the apparatus (Figures 2 and 3) are: (A) The cast-iron cylinder for holding the mixture. (B) Manometer tube used in obtaining the desired mixture. (C) Power pump used in clearing the cylinder of the products of explosion. (D) Exhaust pump for rarifying the air in the cylinder. (E) Indicator. (F) Tuning fork for obtaining a time line on indicator card. (G) Two storage batteries, one wired to terminals in cylinder to give the electric spark for the ignition of the mixture, the other being used in connection with an electro-magnet to keep the tuning fork in vibration while a card is taken; a metallic point attached to the end of one arm of fork tracing a wave line upon the card. (H) Induction coil.

The method of procedure in making an experiment is as follows: The cylinder is thoroughly cleared of the products of explosion by forcing air through it by means of the power pump. The connections between the cylinder and the atmosphere, the cylinder and the power pump, are then closed. To obtain a given mixture, for illustration, one part gas and five parts air, the barometer reading, say, 30 inches; the connections between cylinder exhaust pump and manometer tube are opened, and the exhaust pump is allowed to operate until there is a difference of level of 5 inches of the mercury in the manometer tube; then one-sixth of the original volume of air in the cylinder has been removed. The connection between the cylinder and the exhaust pump is then closed, and just as much gas is allowed to flow into cylinder as air previously removed, bringing the pressure in the cylinder back to atmospheric and also giving the desired mixture, one part gas and five parts air. The connection between the cylinder and the manometer tube is then closed and the gas is allowed to diffuse in the cylinder for a few minutes before ignition. The indicator card is placed on a flat circular disc which is driven from a shaft over the machine. Metallic points are used for the indicator, and on the arm of the tuning fork. The atmospheric line, which in this case is a circle, is taken and a switch is thrown in, causing the tuning fork to vibrate. The indicator cock is opened and time and pressure lines are taken simultaneously upon

the card. The time line in this case is a wave line, from crest to crest corresponding to $\frac{1}{60}$ of a second.

A small projection from the frame carrying the tuning fork enters a spiral groove in a disc which is rotated while the card is taken; the object of this groove being to prevent the wave lines from being superimposed.

The indicator point is kept in contact with the paper for about two seconds. The time of explosion, maximum pressure and pressure for each twelfth of a second, for one second, are measured and plotted upon coördinate paper to scale of ordinates $1'' = 40$ pounds, abscissæ $6'' = 1$ second. Pressures were also measured for each $\frac{1}{60}$ second from time of ignition to maximum pressure, so that the first part of the pressure line could be plotted more accurately. Areas under the curves were obtained by planimeters.

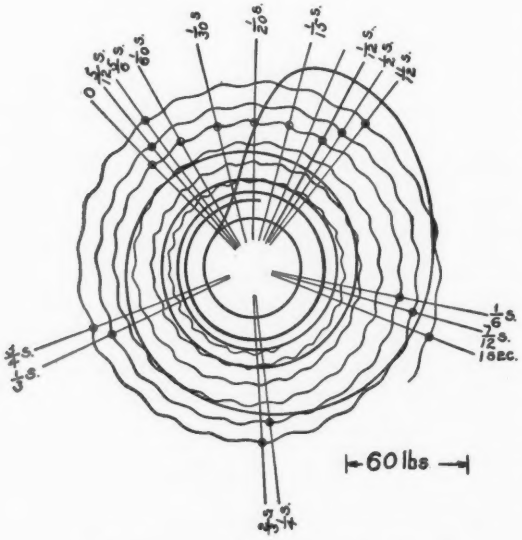

FIG. 4. — A SAMPLE INDICATOR-CARD.

To determine which of the mixtures has the greatest capacity for doing work, suppose 1 cubic inch of illuminating gas to be used in each of the mixtures, 1–3, 1–4, 1–5, etc., then they would measure 4, 5, 6, etc., cubic inches. Let them be placed in cylinders of 4, 5, 6, etc., square inches piston area, the pistons will be displaced 1 inch from the bottom of the cylinder. If the pressure on the piston were the same, equal movements of the piston would give equal power; if, therefore, the mixtures gave equally good results, the mean pressure multiplied by the piston area will in all cases be the same. The

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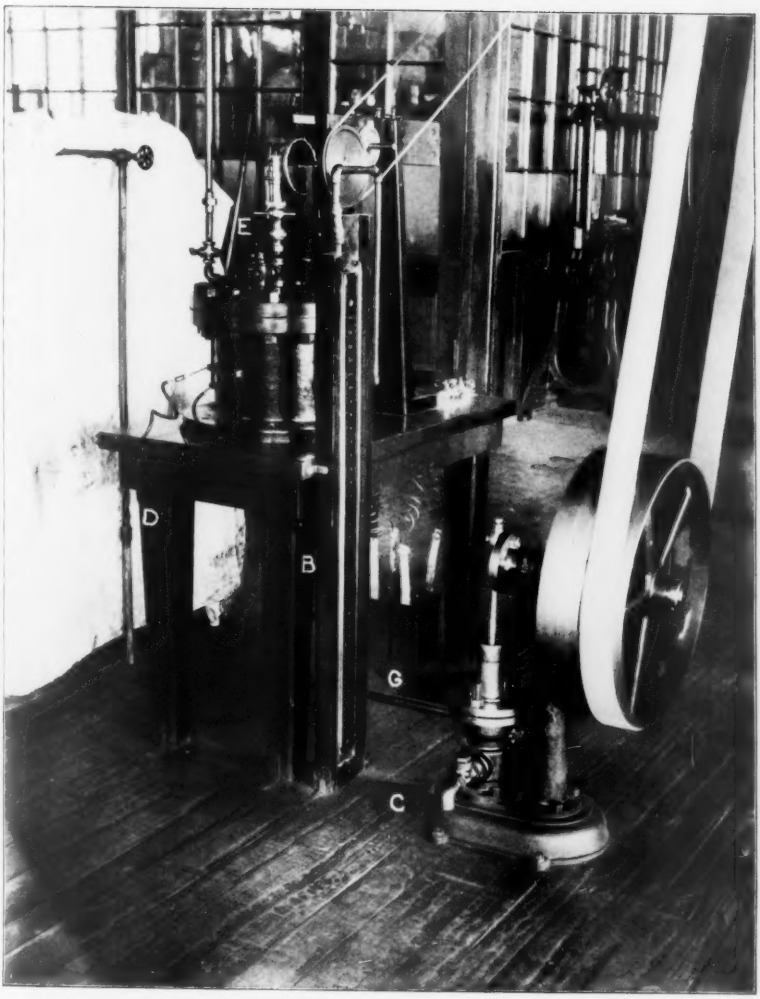

FIG. 3.—THE SAME APPARATUS VIEWED FROM THE OPPOSITE SIDE.

mean pressures were obtained from the plot of the cards on coördinate paper, the same scale being used in all cases.

The first fifth of a second only was considered, for in that time the ordinary gas engine has completed its working stroke. Column 6 of the results of tests shows the relative power of the mixtures. By a similar line of reasoning we obtain column 12 of the table, which shows the relative power of the mixtures to resist cooling. As the method here employed is applicable to other gases, we are enabled to compare the relative value of any similar explosive mixture.

RESULTS OF TESTS ON EXPLOSIVE MIXTURES OF ILLUMINATING GAS AND AIR.

Mixture. (By parts.)	Maximum pressure. (Lbs. per sq. in.)	Time of explosion. (Seconds.)	First ⅕ Second.				⅕ Second after Maximum Pressure.				
			Area. (Square inches.)	Mean pressure. (Lbs. per sq. in.)	Mean pressure. Divided by proportion of gas.	Final pressure.	Area. (Square inches.)	Mean pressure. (Lbs. per sq. in.)	Mean pressure. Divided by proportion of gas.	Final pressure.	Final pressure. Divided by proportion of gas.
1	2	3	4	5	6	7	8	9	10	11	12
Gas-Air.											
1 — 3	45	.49	0.32	11	44	26	1.30	43	172	40	160
1 — 4	86	.08	1.77	59	295	61	1.88	62	310	46	230
1 — 5	96	.05	1.86	62	372	52	1.93	64	384	44	264
1 — 6	88	.05	1.80	60	420	54	1.93	64	448	46	322
1 — 7	86	.06	1.97	66	528	58	1.93	64	512	48	384
1 — 8	87	.06	1.71	57	513	53	1.83	61	549	46	414
1 — 9	77	.08	1.60	53	530	57	1.86	62	620	46	460
1 — 10	71	.11	1.36	45	495	56	1.69	56	616	45	495
1 — 11	68	.14	1.21	40	480	60	1.66	55	660	43	516
1 — 12	39	.33	0.35	12	156	29	0.98	33	429	30	390
1 — 13	32	.42	0.18	6	84	16	0.79	26	364	24	336
1 — 14	9	.42	0.05	2	30	4	0.24	8	120	8	120

Applied Mechanics.

TORSION TESTS ON COPPER WIRE. (Series 1.)

Date, 1897.	Diameter of cross section. (Ins.)	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Oct. 28 . . .	0.1658	12.0	36.6	52.1	40,900	4.34
Oct. 29 . . .	0.1644	12.0	37.3	53.8	42,800	4.48
Oct. 29 . . .	0.1643	12.0	38.3	42.1	43,900	3.51
Nov. 5 . . .	0.1664	12.0	37.6	46.4	41,600	3.87
Nov. 5 . . .	0.1678	12.0	37.4	51.5	40,300	4.29
Nov. 7 . . .	0.1663	12.0	37.0	43.4	40,900	3.62
Nov. 9 . . .	0.1669	12.0	38.0	57.8	41,600	4.82
Nov. 9 . . .	0.1666	12.0	38.0	16.8	41,900	1.40
Nov. 11 . . .	0.1656	12.0	36.5	49.2	41,000	4.10
Nov. 17 . . .	0.1663	12.0	35.7	46.3	39,500	3.87

TORSION TESTS ON COPPER WIRE. (Series 2.)

Date, 1897.	Diameter of cross section. (Ins.)	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws at fracture.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Nov. 17 . . .	0.2586	10.0	130.7	37.7	38,500	3.77
Nov. 17 . . .	0.2584	10.0	126.8	37.8	37,400	3.78
Nov. 17 . . .	0.2582	12.0	118.6	43.2	35,100	3.60
Nov. 18 . . .	0.2576	10 0	116.3	33.1	34,600	3.31
Nov. 18 . . .	0.2572	10.0	127.6	31.5	38,100	3.15
Nov. 19 . . .	0.2575	10.0	122.3	34.2	36,400	3.42
Nov. 20 . . .	0.2578	10.0	125.0	43.2	37,200	4.32
Nov. 20 . . .	0.2581	10.0	131.5	46.5	38,900	4.65
Nov. 22 . . .	0.2579	10.0	124.7	30.8	37,000	3.08

Note. — This material was a very soft ductile grade of copper.

TORSION TESTS ON COPPER WIRE. (Series 3.)

Date, 1897.	Diameter of cross section. (Ins.)	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws at fracture.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Dec. 4 . . .	0.1037	10.0	9.1	28.1	41,400	2.81
Dec. 6 . . .	0.1038	8.0	9.5	29.6	43,300	3.70
Dec. 9 . . .	0.1025	8.0	11.6	24.4	54,700	3.05
Dec. 10 . . .	0.1034	8.0	12.9	5.2	59,400	.65
Dec. 11 . . .	0.1050	8.0	9.7	18.1	42,700	2.26
Dec. 18 . . .	0.1062	8.0	10.3	39.0	43,800	4.88
Dec. 18 . . .	0.1041	8.0	11.5	27.8	51,800	3.48
Dec. 28 . . .	0.1046	10.0	10.6	23.2	47,200	2.32

NOTE. — This material was a hard grade of copper.

TORSION TESTS ON BRASS TUBING.

Date, 1898.	Outside diameter of cross section. (Ins.)	Inside diameter of cross section (Ins.)	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Apr. 7 . . .	0.246	0.186	10.0	81.5	$1\frac{1}{2}$	41,500	.15
Apr. 9 . . .	0.253	0.173	10.0	132.5	$1\frac{3}{4}$	53,300	.18
Apr. 22 . . .	0.253	0.158	10.0	132.0	$\frac{1}{4}$	49,200	.03
Apr. 22 . . .	0.253	0.158	10.0	131.0	$\frac{1}{4}$	48,900	.03
Apr. 22 . . .	0.253	0.158	10.0	138.0	$\frac{1}{4}$	51,300	.03
Apr. 23 . . .	0.253	0.158	10.0	139.0	$\frac{1}{4}$	51,800	.03
Apr. 23 . . .	0.253	.0158	10.0	135.0	$\frac{1}{4}$	50,300	.03
Apr. 25 . . .	0.253	0.158	10.0	134.0	$\frac{1}{4}$	50,000	.03
Apr. 25 . . .	0.253	0.158	10.0	115.0	$\frac{1}{8}$	42,900	.02

TORSION TESTS ON ROUND BRASS ROD.

Date, 1898.	Diameter of cross section.	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Apr. 5 . . .	0.2518	8.00	17.5	12¾	55,800	1.6
Apr. 5 . . .	0.2515	9.00	17.4	15	55,700	1.7
Apr. 5 . . .	0.2518	8.00	18.6	13¼	59,300	1.7
Apr. 5 . . .	0.2512	9.00	17.4	15	55,900	1.7
Apr. 5 . . .	0.2515	9.00	17.8	15½	57,000	1.7
Apr. 7 . . .	0.2510	8.00	17.0	11½	54,600	1.4
Apr. 9 . . .	0.2512	8.00	16.3	14	51,200	1.8
Apr. 12 . . .	0.2511	8.00	16.4	13	52,800	1.6
Apr. 12 . . .	0.2512	8.00	16.4	13	52,700	1.6
Apr. 14 . . .	0.2512	9.00	16.6	15	53,300	1.7
Apr. 14 . . .	0.2508	8.00	17.0	14	54,900	1.8
Apr. 15 . . .	0.2505	8.00	16.6	12	53,800	1.5
Apr. 16 . . .	0.2505	8.00	17.1	13¾	55,400	1.7
Apr. 16 . . .	0.2505	8.00	16.9	14	54,800	1.8
Apr. 25 . . .	0.2512	10.00	16.5	16¼	53,000	1.6

TORSION TESTS ON GALVANIZED IRON WIRE.

Date, 1897.	Diameter of cross section. (Ins.)	Length of specimen between jaws. (Ins.)	Maximum twisting moment. (In. lbs.)	Number of turns between jaws at fracture.	Apparent outside fiber stress. (Lbs. per sq. in.)	Average number of turns per inch.
Oct. 18 . . .	0.1491	12.0	53.3	39.9	81,800	3.33
Oct. 18 . . .	0.1488	12.0	52.5	37.8	81,200	3.15
Oct. 20 . . .	0.1489	12.0	52.8	40.4	81,300	3.37
Oct. 20 . . .	0.1490	12.0	53.3	41.7	82,000	3.48
Oct. 20 . . .	0.1492	12.0	51.7	39.5	79,300	3.29
Oct. 27 . . .	0.1492	12.0	50.9	32.5	78,100	2.71
Oct. 27 . . .	0.1492	12.0	52.4	40.9	80,400	3.41
Oct. 27 . . .	0.1491	12.0	53.1	37.2	81,500	3.10
Oct. 28 . . .	0.1492	12.0	53.3	41.1	81,800	3.43
Nov. 24 . . .	0.191	12.0	107.0	35.4	77,900	3.0
Nov. 24 . . .	0.191	12.0	110.0	36.4	80,100	3.0
Dec. 2 . . .	0.191	10.0	108.9	29.7	80,100	3.0
Dec. 2 . . .	0.191	10.0	105.9	28.6	77,900	2.9
Dec. 4 . . .	0.191	10.0	107.8	27.6	78,600	2.8
Dec. 6 . . .	0.190	10.0	106.4	27.4	78,500	2.7
Dec. 9 . . .	0.190	10.0	106.5	31.7	79,600	3.2
Dec. 10 . . .	0.191	10.0	106.9	29.0	78,400	2.9
Dec. 11 . . .	0.191	10.0	105.0	28.0	76,500	2.8
Dec. 18 . . .	6.191	10.0	116.0	26.8	85,300	2.7
Dec. 20 . . .	0.191	10.0	109.7	30.0	80,700	3.0
Dec. 31 . . .	0.191	12.0	105.6	36.4	77,500	3.0
Dec. 31 . . .	0.191	12.0	103.0	33.9	75,500	2.8
Jan. 1 . . .	0.190	10.0	105.1	30.7	77,600	3.1
Jan. 1 . . .	0.190	10.0	103.5	27.2	76,600	2.7
Jan. 1 . . .	0.191	10.0	105.8	31.6	77,600	3.2
Jan. 1 . . .	0.191	10.0	104.2	28.5	76,400	2.9
Mar. 29 . . .	0.193	10.0	115.0	36.3	81,500	3.6
Mar. 31 . . .	0.193	10.0	120.0	37.5	87,000	3.8
Mar. 31 . . .	0.190	10.0	108.0	35.5	80,200	3.6
Apr. 4 . . .	0.194	10.0	116.0	....	80,900	...
Apr. 4 . . .	0.194	10.0	108.0	38.0	75,100	3.8
Apr. 21 . . .	0.193	10.0	117.0	37.5	82,900	3.7
Apr. 21 . . .	0.193	10.0	123.0	40.8	87,100	4.1

TRANSVERSE STRENGTH OF SPRUCE BEAMS.

Load applied at middle of span.

Number of test.	Width and depth. (Ins.)	Distance between supports. (Feet.)	Limits of loads for calculation of modulus of elasticity.	Deflection for difference of loads used in calculation of modulus of elasticity. (Ins.)	Modulus of elasticity. (Lbs. per sq. in.)	Breaking load. (Lbs.)	Weight of beam. (Lbs.)	Modulus of rupture. (Lbs. per sq. in.)	Maximum intensity of longitudinal shear. (Lbs. per sq. in.)	Manner of Breaking.	Remarks.
580	4 x 12	23.5	500–1500	.6155	1,310,000	5,910	180	4,430	95	Tension.	
581	5⅞ x 12	21	500–4500	1.3662	1,150,000	9,180	315	4,210	101	Tension.	
582	6 x 11⅞	19	500–4500	.7361	1,600,000	14,685	350	6,030	158	Tension.	
583	6 x 12	19	500–4500	.9713	1,200,000	12,880	310	5,300	140	Crushing and tension.	
584	5⅞ x 11⅞	21	1000–4000	.8685	1,404,000	10,790	320	5,050	119	Tension.	
585	6 x 12	19	500–4500	.9586	1,190,000	8,155	335	3,290	88	Tension.	
586	6 x 12	19	500–4500	.9626	1,190,000	12,790	330	5,130	137	Tension.	
587	6 x 12	10	500–6500	.2519	993,000	22,230	155	4,650	233	Longitudinal shear.	Part of beam No. 584.
588	6 x 12	9	500–6500	.1508	1,210,000	27,470	145	5,150	288	Longitudinal shear.	
589	5⅞ x 12	9	.....	.....	.....	21,030	135	4,040	225	Tension.	Part of beam No. 581.
590	5⅞ x 12	8.5	.....	.....	.....	23,930	135	4,340	216	Shearing and tension.	Part of beam No. 583.
591	3⅞ x 11⅞	11	500–2500	.1688	1,120,000	11,110	105	4,270	208	Tension.	
592	3⅛ x 11⅞	11	500–3500	.1917	1,500,000	21,380	100	8,330	371	Tension.	Well seasoned.
593	4⅛ x 12⅛	17	500–2500	.6280	891,000	7,155	225	3,590	110	Tension.	
594	4⅛ x 12⅛	15	500–3500	.4559	1,310,000	12,305	190	5,520	187	Tension.	
595	4 x 12⅛	15	500–4500	.6199	1,320,000	11,810	185	5,460	185	Tension.	
602	4 x 12⅛	15	500–6500	.8346	1,430,000	8,930	185	4,060	139	Tension.	
604	3⅛ x 12⅛	14	500–6500	.688	1,500,000	14,625	155	6,500	237	Crushing and tension.	
605	4 x 12	15	500–6500	.820	1,540,000	12,580	150	5,940	199	Longitudinal shear.	
609	4 x 12	17	500–4500	.858	1,430,000	11,055	190	6,240	185	Tension.	
610	3⅞ x 11⅞	15	500–4500	.636	1,410,000	13,130	150	6,520	216	Crushing and tension.	
611	4 x 12	15	500–4500	.6318	1,340,000	14,025	150	6,610	221	Crushing and tension.	
612	4 x 12	15	500–4500	.5523	1,530,000	13,210	160	6,230	209	Longitudinal shear.	
613	2 x 12	15	500–2000	.5227	1,210,000	4,980	90	4,710	158	Crushing and tension.	
614	1⅞ x 12	14	500–2500	.5273	1,390,000	5,905	95	5,560	200	Crushing and tension.	
615	1⅞ x 12	14	500–2500	.6428	1,140,000	4,200	100	3,970	143	Tension.	

TRANSVERSE TESTS OF YELLOW PINE BEAMS.

Load applied at middle of span.

Number of test.	Width and depth. (Ins.)	Distance between supports. (Feet.)	Limits of loads for calculation of modulus of elasticity.	Deflection for difference of loads used in calculation of modulus of elasticity. (Ins.)	Modulus of elasticity. (Lbs. per sq. in.)	Breaking load. (Lbs.)	Weight of beam. (Lbs.)	Modulus of rupture. (Lbs. per sq. in.)	Maximum intensity of longitudinal shear. (Lbs. per sq. in.)	Manner of Breaking.	Remarks.
596	$4\frac{1}{16} \times 12$	17	500–9500	1.4630	1,860,000	15,615	285	8,240	245	Crushing first, and then longitudinal shear.	
597	$3\frac{7}{8} \times 12$	17	500–6500	.9104	2,090,000	15,005	215	8,290	245	Tension.	
598	$4\frac{1}{8} \times 12\frac{1}{8}$	18	500–3375	.4083	2,410,000	7,475	285	4,070	116	Longitudinal shearing.	
599	4×12	17	500–6500	.8799	2,090,000	14,610	335	7,850	234	Longitudinal shearing.	
600	$4\frac{1}{8} \times 12\frac{1}{4}$	17	500–6500	.7939	2,120,000	17,475	300	8,710	264	Longitudinal shearing.	
601	$4 \times 12\frac{1}{4}$	18	500–3500	.7123	1,360,000	11,480	305	6,030	177	Crushing and tension.	
603	$4\frac{3}{16} \times 12\frac{1}{16}$	14	500–8500	.7277	1,830,000	17,705	245	7,600	275	Tension.	
606	$3\frac{7}{8} \times 12\frac{1}{4}$	17	500–6500	.832	2,160,000	13,430	280	7,330	227	Tension.	
607	$3\frac{3}{4} \times 12\frac{1}{8}$	17	500–4500	.872	1,460,000	7,205	245	4,070	125	Tension.	
608	$4 \times 11\frac{7}{8}$	17.5	500–4500	.895	1,550,000	8,230	265	4,670	134	Longitudinal shearing.	

THE PROTECTION OF STEAM-HEATED SURFACES.

By CHARLES L. NORTON.

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THE investigation, of which this is a partial report, has been undertaken at the request of Mr. Edward Atkinson, and has been pursued during a large part of the years 1896 and 1897, and is still incomplete. The first object sought for was the relative efficiency of several kinds of steam-pipe covering now upon the market. The second object was to ascertain the fire risk attendant upon the use of certain methods and materials for insulation of steam pipes. Third, an attempt was made to show the gain in economy attendant upon the increase of thickness of coverings, and to show also the exact financial return which may be expected from a given outlay for covering steam pipes. Further information is given on many minor matters and conditions effecting the transfer of heat from a steam pipe to the surrounding air.

The method adopted is one which, so far as I know, is original. A piece of steam pipe is heated by electricity from the inside. The amount of electrical energy supplied is measured, and hence the amount of heat furnished is known. If the steam pipe is kept at a constant temperature by a given amount of heat, it is because that amount is just equal to the heat it is losing, for if the supply were not equal to the loss, the temperature would rise or fall. In other words, the heat put into the pipe is just equal to the heat lost from it by radiation, convection, and conduction. By measuring the electrical energy supplied I can determine the heat put in, and hence the heat given out or lost. It must be borne in mind that a given amount of electrical energy always produces the same definite amount of heat, the amount of heat furnished by one electrical unit of energy being known with greater accuracy than the amount of heat given out by a pound of steam in condensing.

The apparatus for making tests by this method comprises several

pieces of steam pipe of different diameters and lengths, heated electrically from within by means of coils of wire in oil. The oil is stirred vigorously, and serves as a very efficient carrier of heat from the wires to the pipes. A brief description of the smallest tester may make the details of the apparatus more easily understood. A section is shown in Figure 1.

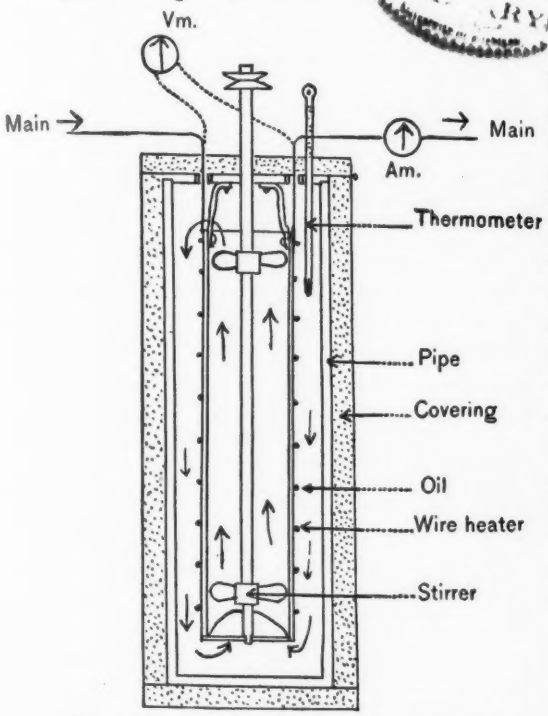

FIG. 1.—PLAN OF THE APPARATUS FOR TESTING STEAM-PIPE COVERINGS.

A piece of 4-inch steam pipe 18 inches long is closed at one end by a plate welded in, and at the other by a tightly fitting cover. This pipe is then filled with cylinder oil, and a coil of wire of sufficient carrying capacity, and a stirrer, are introduced into the oil. A thermometer is inserted in such a position as to record the temperature of the oil. An ammeter and voltmeter or a wattmeter may then be connected so as to record the amount of electrical energy supplied. The stirring must be brisk, and if enough power is put into the stirrer to be comparable with the electrical energy supplied, such amount must of course be added, as it also is converted into heat. It is my custom to suspend the apparatus in the middle of the room on non-conducting cords, and read the thermometer with a telescope, so that no heat from the person of the observer may be added to the supply given to the cover from within, and also that care may be taken not to produce air currents by walking near the apparatus during a test.

In making a test the following operations are carried out, and observations are taken in the following order:

The current is turned on, and heat is generated in the wire coil until the wire, oil, and steam pipe have reached the desired temperature at which it is proposed to test. The current is then gradually diminished, until it is found to be of just the amount necessary to keep the pipe at this temperature without a rise or fall of one-tenth of a degree in 30 minutes. A reading of the voltage and current is now taken at intervals of 30 seconds, and the watts and B. T. U. are computed from their average. We then have the number of B. T. U. lost from the outside of this particular pipe at this particular temperature. If, now, we place a steam-pipe cover around the pipe, we shall find that a less amount of energy is sufficient to keep it at the required temperature, the difference being the amount of heat saved by the covering. The minimum length of time considered sufficient for the equalization of heat, or "soaking in" to the cover, is 6 hours. If, after a second heating of 6 hours, no change in the conducting power is noted, the cover is considered in a permanent condition, and is tested. Some covers, notably those composed wholly or in part of wool, can not be considered dry and constant until after an exposure upon a pipe at 200 pounds pressure for 6 or 8 days. Covers containing sulphate of lime are also slow in drying.

The three thermometers used were frequently standardized in naphthaline, and were examined to note any disagreement among themselves.

A discussion of the position of the tester and its exposure to air currents will be found in a later paragraph.

All tests were made at a temperature corresponding to 200 pounds steam pressure.

A comparative test was made in 1895 upon a number of steam-pipe covers on a 4-inch tester 16 inches long. The results obtained have been published in the circulars issued by the Boston Manufacturers Mutual Fire Insurance Company, and by the Steam Users' Association. The values given were stated to be purely *relative*, the specimen being too small to give reliable data on the *absolute* conduction, and the surrounding conditions not being controlled other than to maintain them constant during the several runs. The ends of the specimen were covered by massive heads, and the whole tester was situated within a few inches of a brick wall and a stone pier. It was

FIG. 2. — GENERAL VIEW OF THE APPARATUS IN POSITION.

called to my attention that the heat loss was probably high, and I agree that the exposure was such as to make it so, being a rather harsh test, but one which was rigidly uniform in its requirements of the several covers. In short, the actual loss of heat per square foot of the pipe surface was correct for that particular piece under the conditions of the test, but was not sufficient for the estimation of the actual saving which might be expected from the general use of coverings. I deemed it wise, therefore, to construct new heaters 4 and 10 inches in diameter, and 36 inches long. These were suspended by non-conducting cords in the center of the laboratory, so as to hang freely and not be in contact with any conducting supports. Conduction up to the lead wires and stirring rod was found to be negligible.

It seems to me that I have approached more nearly the conditions of actual practice than can be obtained by any other method of testing, except the actual use of a long run of pipe, and the determination of the amount of heat put into such a pipe by the "condensation" method offers many difficulties and is open to much uncertainty. I feel, therefore, that in adopting this method I am using a reasonable exposure for the pipe, and have an exceptionally good opportunity to measure the heat supplied.

Results are given in English units, the British Thermal Unit or "pound — degree" being the measure ordinarily used by engineers. Its abbreviated form in the tables is B. T. U., it being the amount of heat necessary to raise 1 pound of water 1° F.

The general appearance of the testing apparatus is shown in Figure 2. Table I gives the relative conductivity of the various kinds of steam-pipe cover tested up to April, 1898.

It gives the results of the tests upon most of the samples used, some being omitted when found to be of such low efficiency as to be of doubtful value.

Specimen A. Nonpareil Cork Standard, consists of granulated cork, pressed in a mould at high temperature and then submitted to a fireproofing process.

Specimen B. Nonpariel Cork Octagonal, is similar in composition, but is made up of several strips of cork, instead of two semi-cylindrical sections.

Specimen C. Manville High Pressure Sectional Cover, is composed of an inner jacket of earthy material and an outer jacket of wool felt, the whole being $1\frac{1}{4}$ inches thick.

TABLE I.

Specimen.	Name.	Maker.	B. T. U. loss per sq. ft. pipe surface per minute.	Per cent. or ratio of loss to loss from bare pipe.	Thickness in inches.	Weight in ounces per ft. of length, 4" diameter.
A	Nonpareil Cork Standard . .	Nonpareil Cork Co. . .	2.20	15.9	1.00	27
B	Nonpareil Cork Octagonal . .	Nonpareil Cork Co. . .	2.38	17.2	.80	16
C	Manville High Pressure . . .	Manville Covering Co. .	2.38	17.2	1.25	54
D	Magnesia	Keasby & Mattison Co. .	2.45	17.7	1.12	35
E	Imperial Asbestos	H. F. Watson	2.49	18.0	1.12	45
F	"W. B."	H. F. Watson	2.62	18.9	1.12	59
G	Asbestos Air Cell	Asbestos Paper Co. . .	2.77	20.0	1.12	35
H	Manville Infusorial Earth . .	Manville Covering Co. .	2.80	20.2	1.50	..
I	Manville Low Pressure . . .	Manville Covering Co. .	2.87	20.7	1.25	..
J	Manville Magnesia Asbestos .	Manville Covering Co. .	2.88	20.8	1.50	65
K	Magnabestos	Keasby & Mattison Co. .	2.91	21.0	1.12	48
L	Moulded Sectional	H. F. Watson	3.00	21.7	1.12	41
O	Asbestos Fire Board	Asbestos Paper Co. . .	3.33	24.1	1.12	35
P	Calcite	Philip Carey Co. . . .	3.61	26.1	1.12	66
	Bare Pipe	.....	13.84	100.	....	..

Specimen D. The Keasby & Mattison Magnesia, is a moulded sectional cover, composed of about 90 per cent. carbonate of magnesia.

Specimen E. The Imperial Asbestos of the H. F. Watson Company, is essentially an air cell cover, being composed of sheets of asbestos paper which has been indented before being laid up, the indentations serving to keep the thin sheets of paper from coming in close contact with one another, thereby causing a considerable amount of air to be held throughout the body of the cover.

Specimen F. The "W. B." covering of the H. F. Watson Company, is composed of a wool felt with a lining of asbestos paper.

Specimen G. The Asbestos Air Cell Cover of the Asbestos Paper Company, is a cover made up of thin sheets of asbestos paper, fluted or corrugated, and stuck together with silicate of sodium.

Specimen H. A plastic covering made by the Manville Company of Infusorial Earth.

Specimen I. The Manville Low Pressure Covering is similar to Specimen F.

Specimen J. A plastic cover made by the Manville Company, and called by them Magnesia-Asbestos. It contains only a slight amount of carbonate of magnesium.

Specimen K. The Magnabestos of the Keasby & Mattison Company is a moulded cover, containing about 45 per cent. of carbonate of magnesia and a considerable percentage of carbonate of calcium.

Specimen L. The Moulded, Sectional Cover of the H. F. Watson Company is composed mainly of sulphate of calcium and some 20 per cent. of magnesium carbonate, and has upon its outer surface a thick sheet of felt board.

Specimen O. The Asbestos Air Cell Fire Board of the Asbestos Paper Company is similar to Specimen G, except that it has larger cells and contains much more silicate of sodium. It is very hard and strong.

Specimen P. The "Calcite," or Asbestos-Magnesia, of the Philip Carey Company, is a sectional, moulded cover, composed mainly of sulphate of calcium. It has an outer layer of felt board.

In regard to the compositions of Specimens C, J, L, and P, I desire to state that I have made no complete analysis, but have satisfied myself that the principal ingredient is sulphate of calcium and *not* carbonate of magnesium. Prospective purchasers of pipe covers should not be misled by names. Since the appearance of Professor Ordway's reports it has been recognized that carbonate of magnesium was of great value as a non-conductor of heat, hence the name "Magnesia" has been applied to a great many covers. It is to be observed that there is no virtue in a name. Asbestos is merely an incombustible material in which air may be entrapped, but when not porous is a good conductor of heat. Magnesia is a most effective non-conductor. This name has been applied to many compounds, of which the greater part consists of carbonate of calcium or of plaster of Paris, materials which are not good as heat retardents. The percentage of magnesium carbonate and plaster of Paris in several moulded, sectional covers is given in Table II.

I have made no investigation of the effect of the raw materials upon the metal of the pipe other than to satisfy myself that the Cork, Magnesia, Air Cell, and Imperial covers cause no corrosion.

TABLE II.

Specimen.	Name	Percentage Composition.	
		$Mg.CO_3$. Carbonate of Magnesium.	$Ca.SO_4$. Sulphate of Calcium.
D	K. & M. Magnesia	80 to 90	3
C	Manville H. P. Lining	Less than 5	65 to 75
L	Watson Moulded	20 to 25	50 to 60
P	Carey Calcite	Less than 5	75
J	Manville Magnesia Asbestos . .	10 to 15	None.

The conditions of testing were reasonably near the conditions of actual practice. The room temperature was kept at 72° F., and the openings into the room were carefully closed. It was found early in the series that variation in the amount of moisture present in the air altered the amount of heat lost from the covers, but no attempt was made to correct this. The error introduced is not greater than 1 per cent.

It was found that the heat loss per square inch of the flat surfaces at the ends of the pipes was less by several per cent. than the loss from the sharply curved sides, and as all pipe covers tested were used to cover both sides and ends, the figures given in the table show a loss less than would be shown were the pipe surface wholly cylindrical, and more than if it were all flat.

The pipes were suspended from the ceiling, as described in an early paragraph, and the circulation of the air about them was due only to their own convection currents. The variation in thickness in different places on the same specimen was considerable, but an average of twenty measurements was taken and results given in the table to the nearest $\frac{1}{8}$ of an inch. Owing to these variations in thickness, the results of a measurement of the efficiency of any one cover cannot be used to predict the efficiency of a second cover of the same make with an accuracy greater than 2 per cent. Two specimens of each make were tested, and in some cases, four, the mean value being given in the table.

Table III gives the saving, in dollars, due to the use of the various covers.

TABLE III.

Specimen.	Name.	Loss per sq. ft. B.T.U. at 200 lbs.	Saving B. T. U. per sq. ft.	Saving per year per 100 sq. ft.
A	Nonpareil Cork Standard	2.20	11.64	$37.80
B	Nonpareil Cork Octagonal	2.38	11.46	37.20
C	Manville Sectional High Pressure .	2.38	11.46	37.20
D	Magnesia	2.45	11.39	36.90
E	Imperial Asbestos	2.49	11.35	36.80
F	"W. B."	2.62	11.22	36.40
G	Asbestos Air Cell	2.77	11.07	36.00
H	Manville Infusorial Earth	2.80	11.04	35.85
I	Manville Low Pressure	2.87	10.97	35.65
J	Manville Magnesia Asbestos . . .	2.88	10.96	35.60
K	Magnabestos	2.91	10.93	33.50
L	Moulded Sectional	3.00	10.84	35.20
O	Asbestos Fire Board	3.33	10.51	34.20
P	Calcite	3.61	10.23	33.24
	Bare Pipe	13.84	....	....

Table IV shows that at the end of ten years the best of the covers tested will have saved $46.00 more than the poorest. The difference between the several covers of the better grade is exceedingly small.

The money saving is computed on the following assumptions: Coal at $4.00 a ton evaporates 10 pounds of water per pound of coal; the pipes are kept hot 10 hours a day, 310 days a year. If computations are made, as is sometimes done, on an assumption that the pipes are hot 24 hours a day, 365 days in a year, the saving is nearly three times that shown in Table III.

Generally speaking, a cover saves heat enough to pay for itself in a little less than a year at 310 ten-hour days, and in about four months at 365 twenty-four-hour days.

It is evident that the decision as to the choice of cover must come from other considerations, as well as from the conductivity.

The question of the ability of a pipe cover to withstand the action of heat for a prolonged period without being destroyed or rendered

TABLE IV.

NET SAVING PER 100 SQUARE FEET.

Specimen.	NAME.	1 year.	2 years.	5 years.	10 years.
A	Nonpareil Cork Standard	$12.80	$50.60	$164.00	$353.00
B	Nonpareil Cork Octagonal	12.20	49.40	161.00	347.00
C	Manville Sectional High Pressure .	12.20	49.40	161.00	347.00
D	Magnesia	11.90	48.80	159.50	344.00
E	Imperial Asbestos	11.80	48.60	159.00	343.00
F	"W. B."	11.40	47.80	157.00	339.00
G	Asbestos Air Cell , ...	11.00	47.00	155.00	335.00
H	Manville Infusorial Earth	10.85	46.70	154.25	333.00
I	Manville Low Pressure	10.65	46.30	153.75	332.00
J	Manville Magnesia Asbestos . . .	10.60	46.20	153.00	331.00
K	Magnabestos	10.50	46.00	152.50	330.00
L	Watson's Moulded Sectional . . .	10.20	45.40	151.00	327.00
O	Asbestos Fire Board	9.20	43.40	146.00	317.00
P	Calcite	8.24	41.48	141.20	307.00
Q	Bare Pipe	....	....	.....	.....

less efficient is of vital importance. The increasing use of cork as an insulator has led to many questions as to its ability to remain "fireproof." I have exposed it to a temperature corresponding to 350 pounds of steam for three months, and to a temperature corresponding to 100 pounds for two years, and can detect no change; and I am satisfied, as well as one can be without the actual experience, that any suspicion of its ability to withstand continued heating is groundless.

The magnesia covering is, of course, unquestionable on this ground, being almost indestructible by heating.

The Imperial Asbestos is also perfectly safe from any fire risks, as is the Air Cell and Fire Board.

The Manville Infusorial Earth, and also the Manville Magnesia-Asbestos are liable to no accident from fire, nor is the Carey Calcite.

It is to those covers, the "W. B." of the Watson Company, and

the Manville Sectional and others which possess a composite structure, that I desire to call attention. I do not consider it safe to put upon a steam pipe, wool, hair felt, or woolen felt in any form. The causes of risk are two: First, the wool may become charred by heat from the pipe and finally ignited. However, this can hardly happen, even on high-pressure pipes, when the thickness of fireproof material (asbestos, magnesia, or whatever it may be) is as great as 1 inch. The second and most serious risk is from the presence in shops or mills of the long tubes of wool, dry as tinder, often connecting one room with another, and ready to flash at the slightest rise in the already too great temperature, I would even insist that the canvas jackets on the covers be fireproof. An accident in my own laboratory has proved the actual danger of these wool felts, and I should not be willing to allow their use again. Their efficiency is high as non-conductors, but not higher than any other perfectly safe covers. If the wool is separated by about 1 inch of fireproof material from the pipe, it is not kept so hot and dry, and the risks from outside ignition is less; but I do not endorse the practice of many engineers in wrapping hair felt outside of a sectional cover. The saving due to this practice is indicated in Table V.

The following assumptions have been made in computing Tables IV, V, and VI: First, that all the covers cost $25.00 per 100 square feet, applied. I realize that this is a high figure, perhaps too high, yet it is not far from the list price of several makers, and any attempt to get a definite price from them revealed a maze of discounts and double discounts, and flexible price lists too intricate for an unitiated mind to travel. In case the saving due to a cover which costs $20.00 instead of $25.00 is desired, the simple addition to the final saving of the $5.00 difference makes the necessary correction.

Secondly, by the advice of the makers, I have made an assumption that the cost is not nearly proportional to the thickness. As the thicker coverings are not now made in great quantities, the actual cost of their manufacture is uncertain.

Inspection of Table V shows the saving due to the use of hair felt outside a standard magnesia cover. In five years 100 square feet of hair felt saves $7.00 more than its cost, and in ten years it saves $20.00 above its cost. The further saving due to a second inch outside the first is $8.00 in ten years. Of course the well-known tendency of hair felt to deteriorate should be considered.

In the case of the Nonpareil cork, increasing the thickness from 1 to 2 inches raises the cost from about $25.00 to $35.00 per 100 square feet, and increases the net saving in five years by $10.00, and by $30.00 in ten years. In other words, the second inch of material in use about pays for itself in two years, while the first pays for itself in about one year. The third inch does not increase the saving even in ten years. The second inch, therefore, more than pays for interest and depreciation, while the third fails to do this.

TABLE V.

VARIATIONS IN THICKNESS, ETC.

SPECIMEN.	Saving in B. T. U. per sq. ft. per minute.	Saving in dollars per 100 sq. ft. per year.	NET SAVING.				Approximate cost.
			1 year.	2 years.	5 years.	10 years.	
Magnesia 1⅛ in. thick	11.62	$37.75	$7.75	$45.50	$159	$347	$30
Magnesia 1⅛ in. thick, and 1 in. of Hair Felt	12.38	40.22	5.22	45.44	166	367	35
Magnesia 1⅛ in. thick, and 2 in. of Hair Felt	12.77	41.50	1.50	43.00	167	375	40
Nonpareil Cork:							
1 inch	11.64	37.80	12.80	50.00	164	353	25
2 inch	12.84	41.75	.....	48.50	174	383	35
3 inch	12.94	42.05	.....	34.10	160	370	50
Fire Board:							
1 inch	10.54	34.20	9.20	43.40	146	317	25
2 inch	11.48	37.25	2.25	39.50	151	337	35
3 inch	11.70	38.00	12.00	26.00	140	330	50
4 inch	11.83	38.40	26.60	11.80	127	319	65

In the case of the Asbestos Fire Board, a second inch in thickness causes a saving of $20.00 in ten years, the third and fourth inches showing a loss.

In general it may be said, therefore, that if five years is the length of life of a cover, 1 inch is the most economical thickness, while a cover which has a life of ten years may to advantage be made 2 inches thick.

In view of the custom which prevails to some extent of wrapping asbestos paper around a pipe and surrounding the whole with hair felt, I made tests as to the temperature of the bounding line of the asbestos paper and hair felt, using a LeChatelier Thermo-electric Pyrome-

ter for this purpose. The different samples of asbestos paper give widely varying results, but a general idea of the protection afforded by the paper may be had from Table VI.

TABLE VI.

PROTECTION AFFORDED BY ASBESTOS PAPER. PIPE AT 200 POUNDS PRESS.

Thickness of Asbestos Paper.	Temperature of Pipe.	Temperature of Inside of Hair Felt.	Pressure Corresponding to the Temperature of the Inside of the Hair Felt.
$\frac{1}{64}$ inch.	384.7° F.	356° F.	146 pounds.
$\frac{1}{32}$ inch.	385.0° F.	329° F.	102 pounds.
$\frac{1}{16}$ inch.	384.6° F.	302° F.	70 pounds.
$\frac{1}{8}$ inch.	384.7° F.	266° F.	39 pounds.

I have had my attention called to the varying loss from bare pipes when their surfaces were in varying conditions as regard rust, dirt, paint, etc. I therefore made a few brief tests to satisfy my mind as to the chance of their being any large variation which might influence my figure for the loss from bare pipe, viz., 13.84 B. T. U. per square feet per minute. The results are shown in Table VII.

TABLE VII.

LOSS OF HEAT AT 200 POUNDS FROM BARE PIPE.

CONDITION OF SPECIMEN.	B. T. U. lost per sq. ft. per minute.
New pipe	11.96
Fair condition	13.84
Rusty and black	14.20
Cleaned with caustic potash inside and out	13.85
Painted dull white	14.30
Painted glossy white	12.02
Cleaned with potash again	13.84
Coated with cylinder oil	13.90
Painted dull black	14.40
Painted glossy black	12.10

The rate of heat loss from a bare pipe is also affected by the air circulation and the temperature of the surrounding bodies. A few tests

were made to indicate the magnitude of the errors likely to be caused by variation in these conditions, and a brief examination of some of the results may be interesting. They are given in Table VIII.

TABLE VIII.

EFFECT OF SURROUNDINGS.

CONDITION AND POSITION OF PIPE.	B. T. U. lost per sq. ft. per minute at 200 pounds.
1. Standard condition: hung in center of room .	13.84
2. Near brick wall, between windows	14.26
3. Hung horizontally in center of room . . .	12.06
4. Vertical 10-inch pipe { 36 inches long . . .	13.48
{ 18 inches long . . .	14.42
5. Vertical 18-inch long { 10-inch diameter . .	14.42
{ 4-inch diameter . .	15.20
6. 4-inch diameter in draft from electric fan . .	20.10

Table IX shows the varying loss from a bare pipe with the change in pressure.

TABLE IX.

VARIATION OF HEAT LOSS WITH PRESSURE.

PRESSURE.	Bare Pipe loss B. T. U. per sq. ft. per minute.
340	15.97
200	13.84
100	8.92
80	8.04
60	7.00
40	5.74

A very thorough test was made of the common method of judging a pipe cover by the sensation of warmth given the hand on touching it, and nothing too harsh can be said of this practice. The sensation is dependent to such an extent upon the *nature of the surface* that it fails utterly to give any idea of the actual temperature. I have been unable to devise any method of so attaching a mercury thermometer to the outside of a steam-pipe cover as to make use of it as a testing device in measuring heat loss.

I am desirous of calling attention to the advantages arising from the use of plastic, rather than sectional covers. The case of removal

for repairs or alterations makes the sectional cover better for some work, but there is much pipe surface which might be covered securely with plastic, where a sectional cover is soon ruined by vibration. Of course, the plastic covers offer no possibility of leaky joints and long cracks. It should be borne in mind that in most cases about 20 per cent. of the entire surface to be covered is irregular, and must be covered by plastic or fittings. It will be well for prospective purchasers of pipe cover to see to it that their contracts call for fittings and plastic of as high an efficiency as the sectional cover shows.

I am now testing a considerable number of samples of non-conducting material, not perhaps, classed as pipe covers, but used for heat insulation. Table X gives some figures concerning them which may be of interest.

TABLE X.

MISCELLANEOUS SUBSTANCES.

SPECIMEN.	B. T. U. per sq. ft. per minute at 200 pounds.	Saving in 1 year per 100 sq. ft. pipe.
Box A:		
1. With sand	3.18	$34.60
2. With cork, powdered	1.75	39.40
3. With cork and infusorial earth	1.90	38.90
4. With sawdust	2.15	37.90
5. With charcoal	2.00	38.50
6. With ashes	2.46	36.90
Brick wall 4 inches thick	5.17	28.80
Pine wood 1 inch thick	3.56	33.80
Hair felt 1 inch thick	2.51	36.80
Cabot's seaweed quilt 1 inch thick	2.78	35.90
Spruce 1 inch thick	3.40	33.90
Spruce 2 inches thick	2.31	37.50
Spruce 3 inches thick	2.02	38.50
Oak 1 inch thick	3.65	33.10
Hard pine 1 inch thick	3.72	32.90
Eider-down 1 inch thick, loose	*1.90 to 2.70	.....
Eider-down 1 inch thick, tightly packed	*1.70 to 1.80	.....

*Variable.

The box *A*, referred to in the table, is a $\frac{7}{8}$-inch pine box, large enough to surround the pipe, leaving a 1-inch minimum space at its four sides. In it were tested several materials which I find are used in just this way for steam and cold storage insulation.

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